Introduction to Computational Science & Engineering (CSE)

16.0002 / 18.0002

New and improved !

Now in person !!

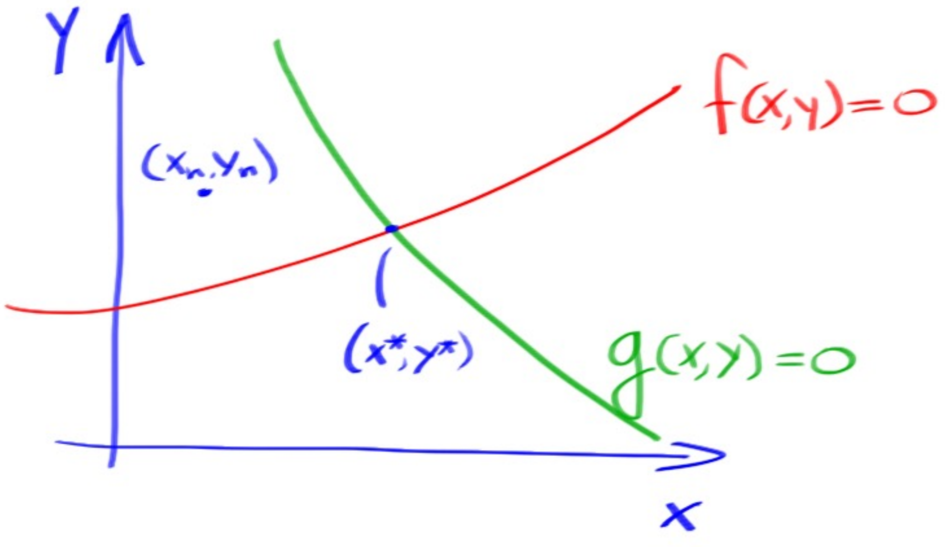
Lecture 6:
Stiffness and implicit methods

Laurent Demanet (Math/EAPS)
Youssef Marzouk (AeroAstro)

10 November 2021

Newton's method in several variables.

$$\begin{bmatrix} f(x,y) \\ g(x,y) \end{bmatrix} = 0$$



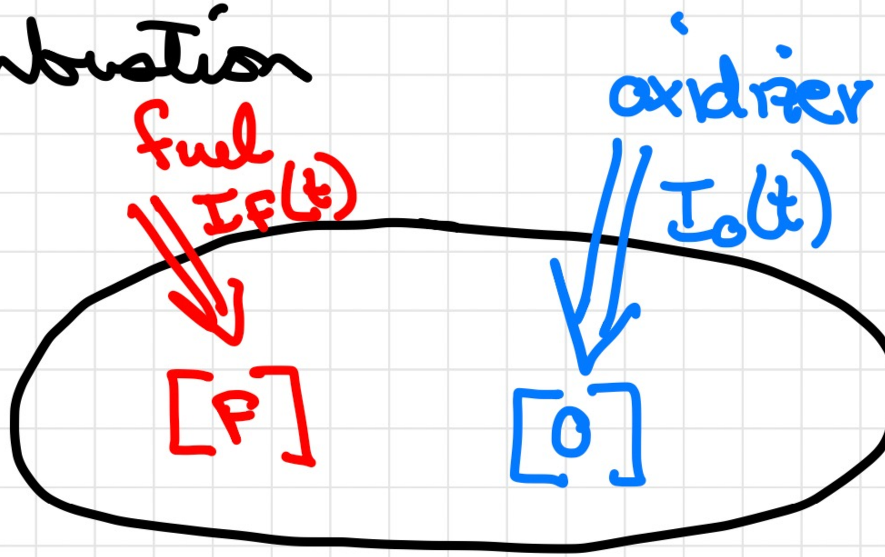
$$\text{Lin}(x,y) = \begin{bmatrix} f_n + \frac{\partial f}{\partial x}(x-x_n) + \frac{\partial f}{\partial y}(y-y_n) \\ g_n + \frac{\partial g}{\partial x}(x-x_n) + \frac{\partial g}{\partial y}(y-y_n) \end{bmatrix}$$

$$= \begin{bmatrix} f_n \\ g_n \end{bmatrix} + J \begin{bmatrix} x-x_n \\ y-y_n \end{bmatrix} = 0 \quad \text{at} \quad \begin{matrix} x = x_{n+1} \\ y = y_{n+1} \end{matrix}$$

J → Jacobian

→ Solve system!
(Gaussian Elim.)

Oscillating Combustion



$$\frac{d[F]}{dt} = -\frac{1}{\tau}[F] + I_F(t)$$

$I_F(t)$ ← oscillations ~10 Hz

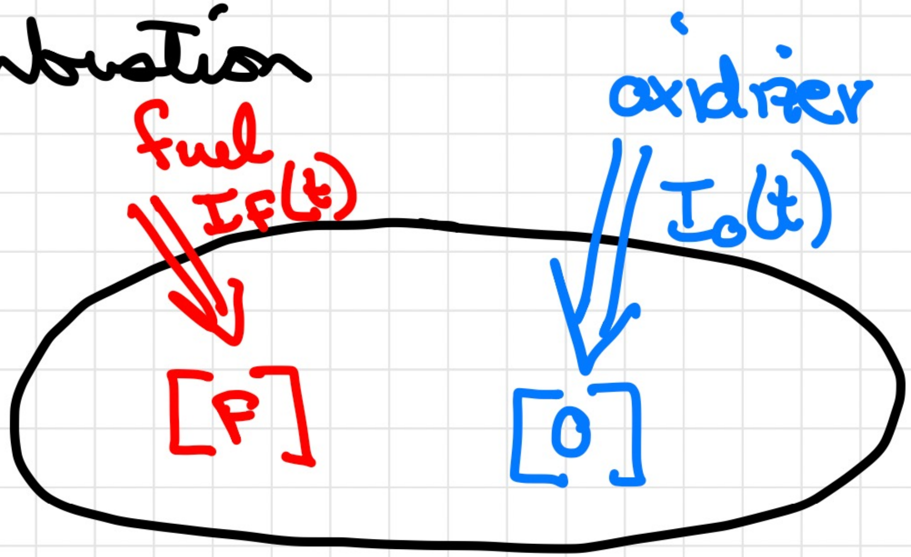
$\tau = 5 \times 10^{-8}\,s$

$$I_F(t) = A_F \frac{1}{2}\left[1 - \cos\left(2\pi \frac{t}{T_F}\right)\right]$$

A_F ← amplitude

$T_F = 0.1\,s$

Oscillating Combustion



$$\frac{d[F]}{dt} = -\frac{1}{\tau}[F] + I_F(t)$$

I_F ← oscillations ~10 Hz

$\tau = 5\times10^{-8}$ s

$$I_F(t) = A_F \, \frac{1}{2}\left[1-\cos\left(2\pi \frac{t}{T_F}\right)\right]$$

A_F ↑ amplitude

T_F ↑ $T_F = 0.1$s

$\frac{T_F}{\tau} = 2\times10^{6}$ ⟸ stiffness measure

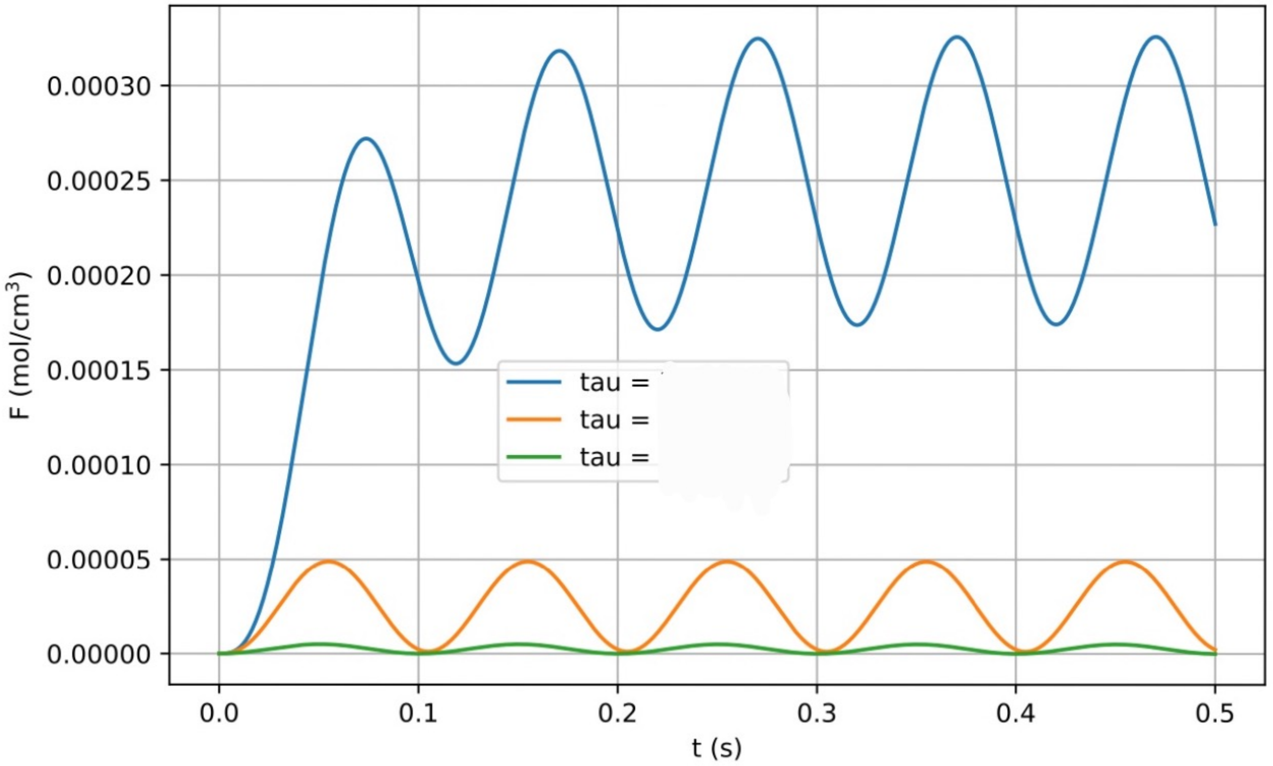


Figure 5.3: Impact of combustion timescale τ on oscillating combustion with $T_F = 0.1\,\mathrm{s}$, and $A_F = 0.01\,\mathrm{mol/cm^3/s}$

Select the correct τ:

(A)
- (blue) $\tau = 5\times 10^{-2}$
- (orange) $\tau = 5\times 10^{-3}$
- (green) $\tau = 5\times 10^{-4}$

(B)
- (blue) $\tau = 5\times 10^{-4}$
- (orange) $\tau = 5\times 10^{-3}$
- (green) $\tau = 5\times 10^{-2}$

(C) None of the above

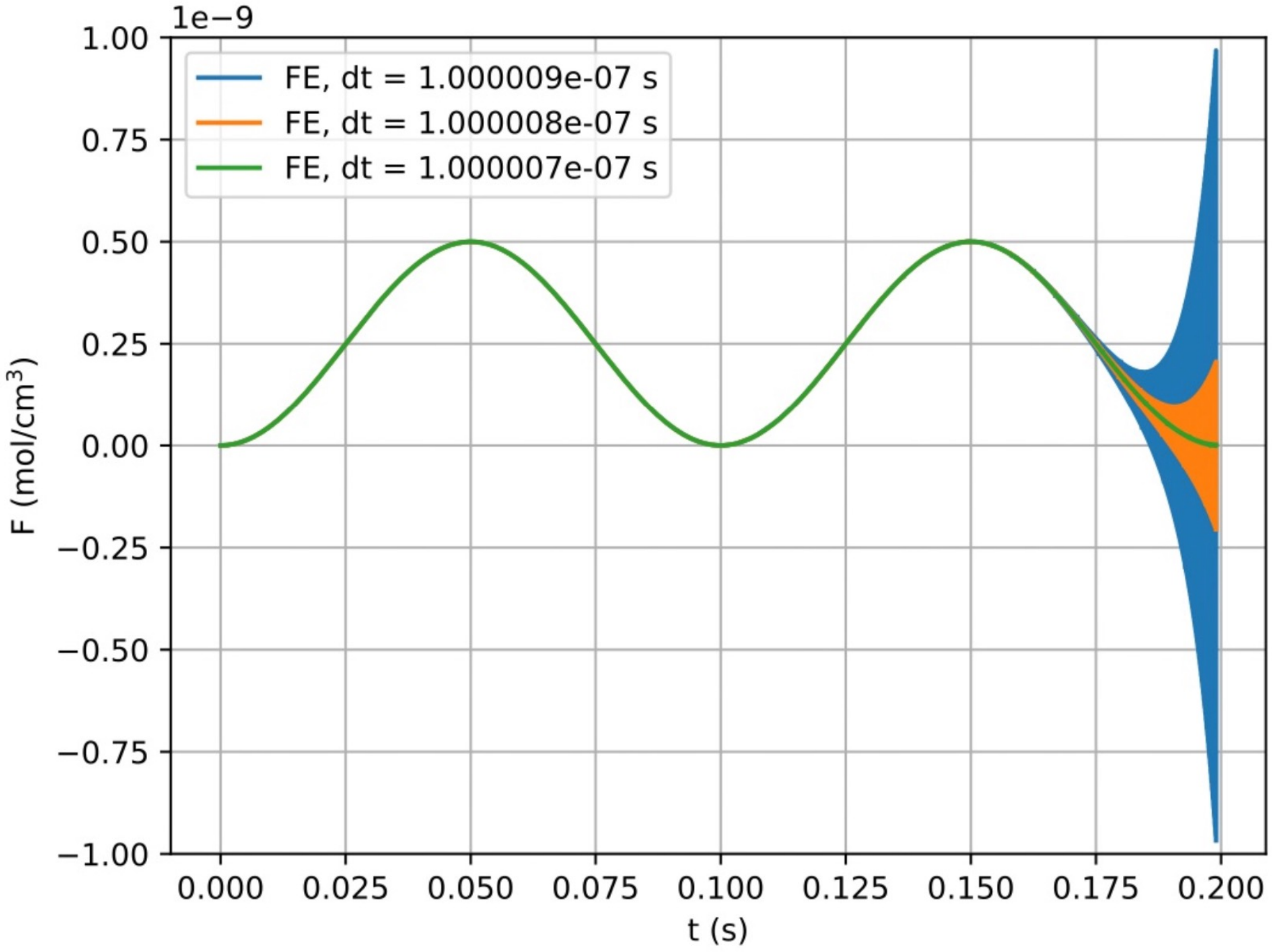


Figure 5.4: Impact of Δt choice using Forward Euler (FE) method to simulate oscillating combustion with $\tau = 5\text{E-}8\,\text{s}$, $T_F = 0.1\,\text{s}$, and $A_F = 0.01\,\text{mol/cm}^3\text{/s}$