Introduction to Computational Science & Engineering (CSE)

16.0002 / 18.0002

New and improved !
Now in person !!

Lecture 7: Optimization I

Laurent Demanet (Math/EAPS)
Youssef Marzouk (AeroAstro)

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Today:

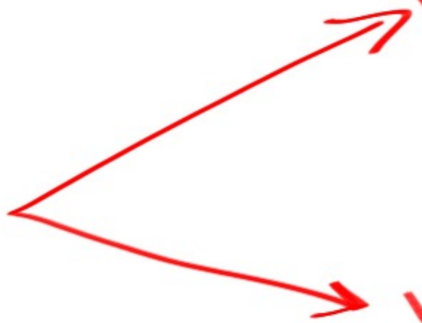
Calculus in the computer.

ind. | Taylor
Finite differences.

Optimization, 1D example

"Find a function most equal to its derivative"

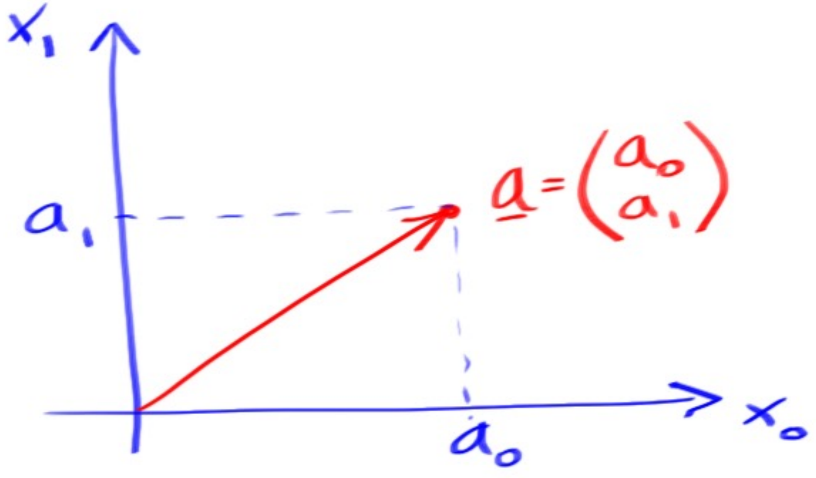
Dot product.



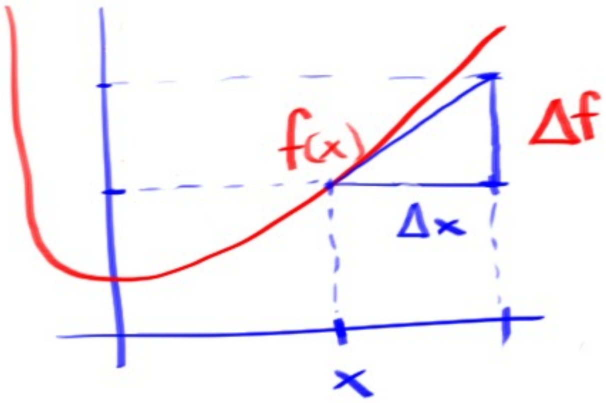
$$\underline{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

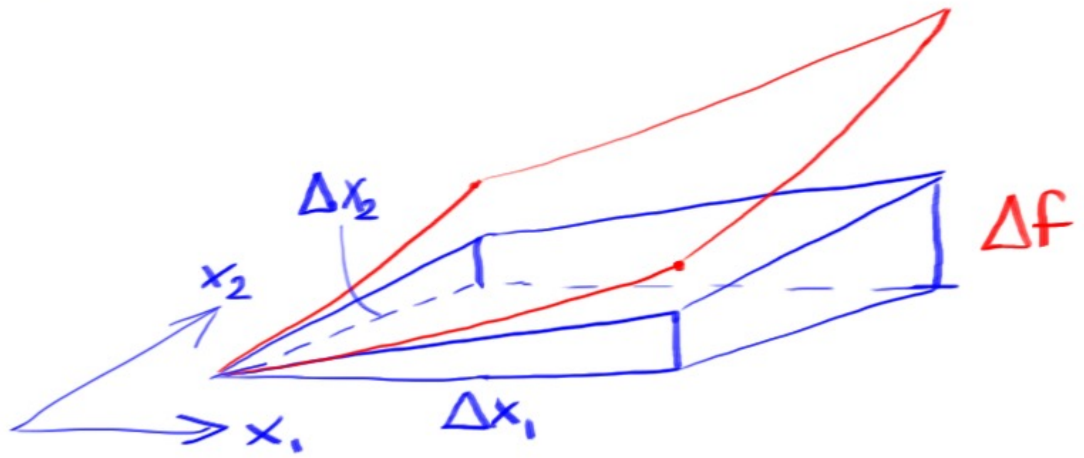
Length / norm



Taylor / Linearization

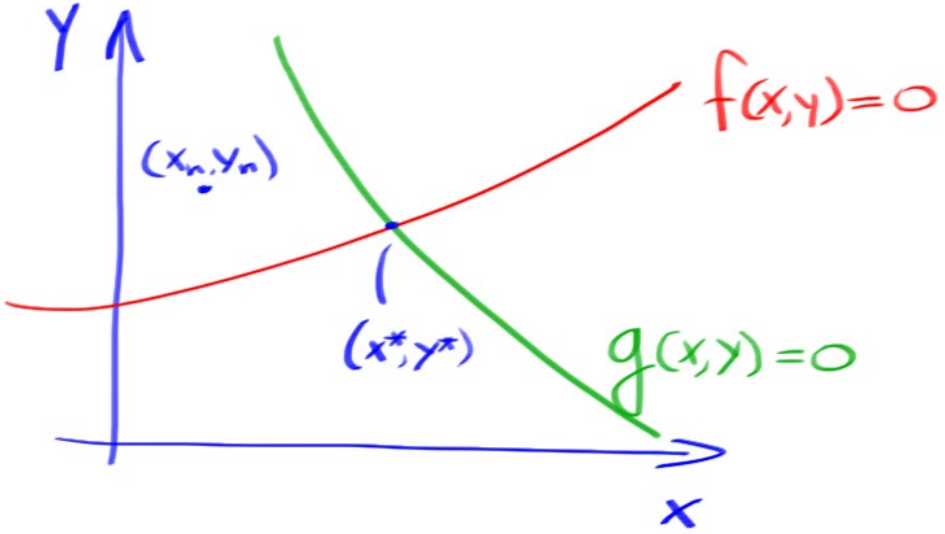


Taylor / Linearization



Newton's method in several variables.

$$\begin{bmatrix} f(x,y) \\ g(x,y) \end{bmatrix} = 0$$

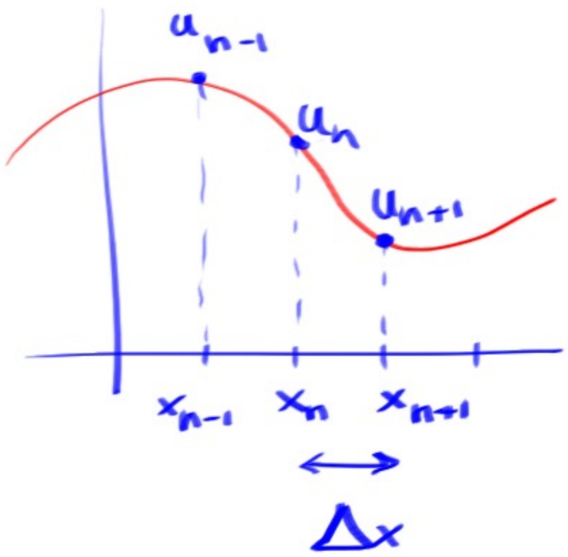


$$\mathrm{Lin}(x,y) = \begin{bmatrix} f_n + \frac{\partial f}{\partial x}(x-x_n) + \frac{\partial f}{\partial y}(y-y_n) \\ g_n + \frac{\partial g}{\partial x}(x-x_n) + \frac{\partial g}{\partial y}(y-y_n) \end{bmatrix}$$

$$= \begin{bmatrix} f_n \\ g_n \end{bmatrix} + \underbrace{J}_{\text{Jacobian}} \begin{bmatrix} x-x_n \\ y-y_n \end{bmatrix} = 0 \quad \text{at} \quad \begin{matrix} x = x_{n+1} \\ y = y_{n+1} \end{matrix}$$

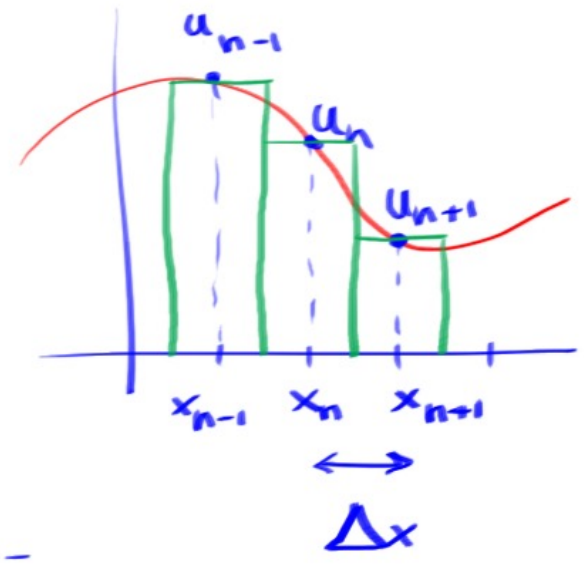
→ Solve system! (Gaussian Elim.)

Finite differences



Quadrature

$$\int f(x)\,dx \simeq$$



$$\cdots + \underbrace{u_{n-1}\,\Delta x} + \underbrace{u_n\,\Delta x} + \underbrace{u_{n+1}\,\Delta x} + \cdots$$

Optimization

Example: find a function $f(x)$ such that

$$f'(x) = f(x), \quad f(0) = 1$$

- Among polynomials?

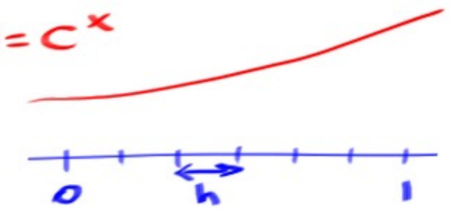
  $x^n \to n x^{n-1}$ always smaller as $x \to \infty$

- Among power laws?

  $f(x) = c^x$

OK ... but which c?

$f(x) = c^x$



$x_n = nh$ $\qquad n = 0, 1, 2, \ldots, N-1$

Compare c^{x_n} to $\dfrac{c^{x_{n+1}} - c^{x_n}}{x_{n+1} - x_n}$ $\qquad (x_{n+1} - x_n = h)$