Introduction to Computational Science & Engineering (CSE)

16.0002 / 18.0002 / CSE.01

Lecture 3:
Linear systems

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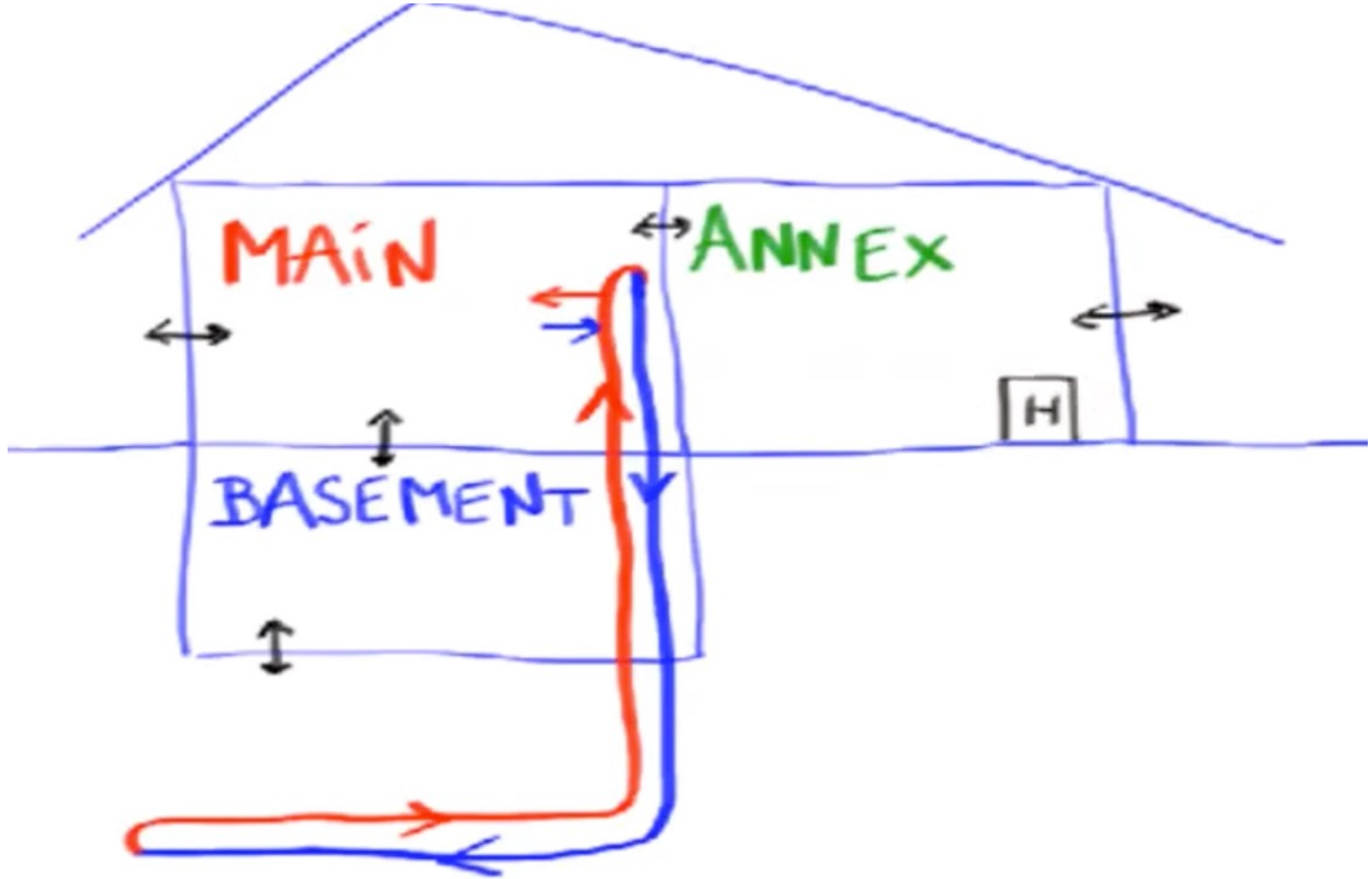
Cooling a cup of coffee

Today: Linear Systems

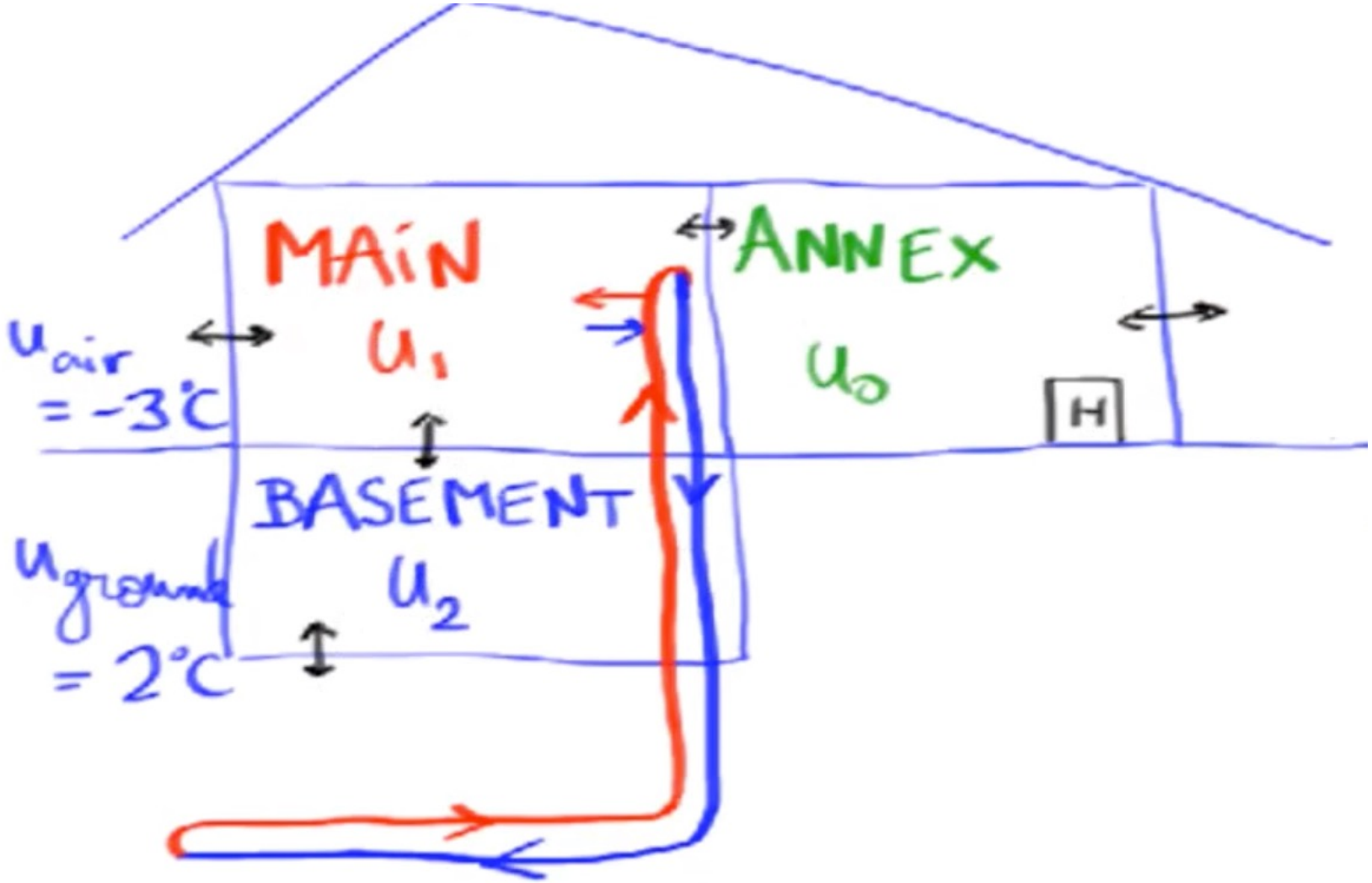
Model	Geothermal home heating
IVP Algorithm	Forward Euler
Equilibrium Algorithm	Gaussian Elimination

→ How to decarbonize home heating?

A. Insulation B. Heat pump
C. Renewable elec. D. Batteries/Storage
E. All of the above !!



GROUND-SOURCE
HEAT PUMP



GROUND-SOURCE
HEAT PUMP

$u_{deep} = 10°C$

$$\text{IVP}\begin{cases} \frac{1}{k}\frac{du_0}{dt} = -2u_0 + u_1 + u_{air} + \frac{1}{k}\frac{1}{mc}P_{heater} & u_0(0) = u_{air} \\ \frac{1}{k}\frac{du_1}{dt} = u_0 - 3u_1 + u_2 + u_{air} + \frac{3}{k}\frac{1}{mc}P_{GSHP} & u_1(0) = u_{air} \\ \frac{1}{k}\frac{du_2}{dt} = u_1 - 2u_2 + u_{ground} & u_2(0) = u_{ground} \end{cases}$$

$$
\text{IVP}\begin{cases}
\frac{1}{k}\frac{du_0}{dt} = -2u_0 + u_1 + \underbrace{u_{air} + \frac{1}{k}\frac{1}{mc}P_{heater}}_{b_0} & u_0(0) = u_{air} \\
\frac{1}{k}\frac{du_1}{dt} = u_0 - 3u_1 + u_2 + \underbrace{u_{air} + \frac{3}{k}\frac{1}{mc}P_{GSHP}}_{b_1} & u_1(0) = u_{air} \\
\frac{1}{k}\frac{du_2}{dt} = u_1 - 2u_2 + \underbrace{u_{ground}}_{b_2} & u_2(0) = u_{ground}
\end{cases}
$$

IVP

$$\begin{cases} \frac{1}{k}\frac{du_0}{dt} = -2u_0 + u_1 + \underbrace{u_{air} + \frac{1}{k}\frac{1}{mc}P_{heater}}_{b_0} & u_0(0) = u_{air} \\ \frac{1}{k}\frac{du_1}{dt} = u_0 - 3u_1 + u_2 + \underbrace{u_{air} + \frac{3}{k}\frac{1}{mc}P_{GSHP}}_{b_1} & u_1(0) = u_{air} \\ \frac{1}{k}\frac{du_2}{dt} = u_1 - 2u_2 + \underbrace{u_{ground}}_{b_2} & u_2(0) = u_{ground} \end{cases}$$

Eq.E

$$\begin{cases} 0 = -2u_0 + u_1 + b_0 \\ 0 = u_0 - 3u_1 + u_2 + b_1 \\ 0 = u_1 - 2u_2 + b_2 \end{cases}$$

Ex. Skydiving

$$\frac{dV}{dt} = g - \frac{\rho_a V^2 A C_D}{m}$$

Ex. Coffee cooling

$$\frac{dT}{dt} = \frac{hA}{mc}(T_{out} - T)$$

Ex. Rabbits - foxes

$$\frac{dr}{dt} = ar - brf$$

$$\frac{df}{dt} = -mf + cbrf$$

What kind of system/eq. is it?

A. Linear, Diff. eq.
B. Nonlinear, Diff. eq.
C. Linear, Algebraic
D. Nonlinear, Algebraic
E. Neither of those

Ex. Find x from $x^2 = 2$

Ex. Find x, y from $\begin{cases} 4x + 5y = 0 \\ -2x + y = 1 \end{cases}$