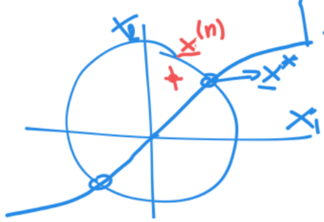
• Newton's method for $f_i(x_1,\dots,x_n)=0$ $\quad i=1,\dots,n$

$$\underline{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

ex. $\left\{\begin{array}{l} x_1^2+x_2^2=1 \\ x_2=\sin(x_1) \end{array}\right.$



Taylor of f_i about $\underline{x}^{(n)}$:

$\underline{x}^{(n+1)}$

$$\boxed{0} = f_i(\underline{x}^*) = \boxed{f_i(\underline{x}^{(n)}) + \underline{\nabla} f_i(\underline{x}^{(n)}) \cdot (\underline{x}^* - \underline{x}^{(n)})} + \cancel{O(\|\underline{x}^* - \underline{x}^{(n)}\|^2)} \qquad i=1,\dots,n$$

$$\underline{\nabla} f_i \cdot \underline{y} = \sum_{j=1}^{n} \frac{\partial f_i}{\partial x_j} y_j \qquad \underline{y} = \underline{x}^* - \underline{x}^{(n)}$$

$$-f_i(\underline{x}^{(n)}) = \sum_{j=1}^{n} \left[\frac{\partial f_i}{\partial x_j}(\underline{x}^{(n)})\right] \left(x_j^{(n+1)} - x_j^{(n)}\right)$$

$\leftarrow J_{ij}$ Jacobian matrix

n unknowns $x_j^{(n+1)}$ $\quad j=1,\dots,n$

n equations $\quad i=1,\dots,n$

need J invertible $\leftarrow$

$$\boxed{\underline{x}^{(n+1)} = \underline{x}^{(n)} - J^{-1} \underline{f}(\underline{x}^{(n)})}$$