

Extremal points are roots of  $\nabla F(x) \leftarrow f(x)=0$

• Newton's method :  $\underline{x}^{(n+1)} = \underline{x}^{(n)} - J^{-1} \underline{f}(\underline{x}^{(n)})$

Here:

$$\underline{x}^{(n+1)} = \underline{x}^{(n)} - \underline{H}^{-1} \nabla F(\underline{x}^{(n)})$$

Shortcut for

GAUSSIAN  
ELIMINATION

$$J = \nabla f \\ = \nabla \nabla F = H \\ (\text{Hessian})$$

$$\rightarrow \underbrace{H}_{A} (\underbrace{\underline{x}^{(n+1)} - \underline{x}^{(n)}}_x) = \underbrace{-\nabla F(\underline{x}^{(n)})}_b$$

• Gradient descent:  $\underline{x}^{(n+1)} = \underline{x}^{(n)} - \alpha \nabla F(\underline{x}^{(n)})$   
 ↗ Step size  $\alpha > 0$   
 "line search" ↗

$$\left. \begin{array}{l} Ax=b \\ \boxed{\begin{array}{c} \text{GE} \\ \text{or} \\ \text{LU} \end{array}} = \boxed{\quad} \\ \text{get } x \\ x = A^{-1}b \end{array} \right\}$$