24.903 Week #2 – 2022-02-07 + 2022-02-09

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> “The primary function of schools is to impart enough facts to make children stop asking questions. Some, with whom the schools do not succeed, become scientists.” (Knut Schmidt-Nielsen)

1 Consolidating and precisifying the system

1.1 Basic types

The ontology or type system we’re assuming has three basic types of things:

- type e: entities
- type s: possible worlds
- type t: truth-values, $\{0, 1\}$

For each of the types, we have a domain of things of that type:

- D_e = the set of all entities
- D_s = the set of all possible worlds (also called W sometimes)
- D_t = the set of truth-values = $\{0, 1\}$

1.2 Functions

Beyond the basic types, we have an infinite set of complex functional types.

For any types α and β, there is the type $\langle\alpha, \beta\rangle$. The domain of things of type $\langle\alpha, \beta\rangle$ is $D_{\langle\alpha,\beta\rangle}$ = the set of all functions from D_α to D_β.

Example: the type $\langle s, \langle e, t\rangle\rangle$ is the type of functions from worlds to functions from entities to truth-values. The meaning of *doctor* is of this type.

1.3 Intensions and Extensions

If a functional type “starts with s”, we call it an intension. The meaning of *doctor* is an intension. If such a function is applied to a world w, we get what is called its “extension at w”.

Example: if you apply the meaning of *doctor* to the world w, you get the extension of *doctor* at w: the set of entities that are doctors in w.

Customarily when we specify the type of an expression, we give the type of its extension. So, even though the meaning of *doctor* is a function from worlds to functions from individuals to truth-values (that is, $[\![\text{doctor}]\!]$ is of type $\langle s, \langle e, t\rangle\rangle$), we usually say that *doctor* is of type $\langle e, t\rangle$.

1.4 The lambda-notation

(Some of this section is adopted from notes entitled “On sets and functions” by my colleague Roger Schwarzschild.)

Here’s a way to write functions you may be familiar with from math:

(1) The function f that maps any natural number to its square:
$f\colon \mathbb{N} \to \mathbb{N}$ such that $\forall n \in \mathbb{N}\colon f(n) = n^2$

There is a notation for describing functions that is inspired by the lambda calculus used in logic and computer science. That notation offers a compact way of describing functions and is widely adopted in linguistics. The specific version we use was introduced by Heim & Kratzer 1998.

In this notation, function names are constructed using the Greek letter lambda ‘λ’ next to a variable, that is followed by a colon and then an expression stating what kind of things the function applies to and that is followed by a period and an expression giving the output of the function.

Our function from (1) can now be written as:

(2) $\lambda n\colon n \in \mathbb{N}.\ n^2$

Here's another example:

(3) $\lambda x\colon\ x$ is a state in the United States . the capital of x

Given the statement after the colon, the function named in (3) will only apply to one of the United States. When that function applies to a state, the result is the capital city of that state.

The "domain condition" that appears after the colon will in our case often consist entirely of a type-specification. An identity function that maps entities to themselves will for example look like this:

(4) $\lambda x\colon\ x \in D_e.\ x$

We will feel free to abbreviate such type requirements by simply notating the variable with its type in a subscript, so the identity function on entities in (4) can also be written as follows:

(5) $\lambda x_e.\ x$

The meanings we have given sentences are functions from worlds to truth-values. We could write such functions as follows:

(6) $[\![$Aïda tabib$]\!] = \lambda w.$ 1 iff Aïda is a doctor in w

We introduce one more convention: when the period is followed by a meta-language statement, we understand the value to be 1 if the statement is true and 0 if the statement is false. So, we rewrite (6) as follows:

(7) $[\![$Aïda tabib$]\!] = \lambda w.$ Aïda is a doctor in w

1.5 Functions with more than one argument

Our meaning for *doctor* is a function that takes a world and an individual and returns the truth-value 1 iff that individual is a doctor in that world.

How does a function take two arguments? It doesn't as such. There are two options: (i) the function takes the ordered pair of the two arguments as its input, (ii) the function takes one argument and then returns a function that takes the second argument and then gives the end result. This latter option was discovered by Moses Schönfinkel and Haskell B. Curry. The idea is often called "currying" (although Heim & Kratzer say it should be called "schönfinkeling").

Here's then how we would write the curried meaning for *doctor*:

(8) $[\![\text{doctor}]\!] = \lambda w_s. (\lambda x_e. x \text{ is a doctor in } w)$

The notation in (8) obscures the fact that in our system the world argument is not fed in by a fellow constituent. In the literature, one often finds the following formulation:

(9) For any world w, $[\![\text{doctor}]\!]^w = \lambda x_e. x \text{ is a doctor in } w$

1.6 Reminder: Composition via Function Application

Here as well, we are feeding w in to make things clearer:

(10) Function Application
If a constituent α has two daughters β and γ, and for any world w, if $[\![\beta]\!]^w$ is a function whose domain contains $[\![\gamma]\!]^w$, then $[\![\alpha]\!]^w = [\![\beta]\!]^w([\![\gamma]\!]^w)$.

1.7 A full calculation

Lexical entries:

(11) For any world w,

a. $[\![\text{Aïda}]\!]^w = \text{Aïda}$

b. $[\![\text{tabib}]\!]^w = \lambda x_e. x \text{ is a doctor in } w$

Calculation of the meaning of the sentence *Aïda tabib*:

(12) For any world w:

$$\begin{aligned}
[\![\text{Aïda tabib}]\!]^w &= [\![\text{tabib}]\!]^w([\![\text{Aïda}]\!]^w) \\
&= (\lambda x_e. x \text{ is a doctor in } w)([\![\text{Aïda}]\!]^w) \\
&= (\lambda x_e. x \text{ is a doctor in } w)(\text{Aïda}) \\
&= 1 \text{ iff Aïda is a doctor in } w
\end{aligned}$$

2 More words and sentences

2.1 Lots of predicates of type $\langle e, t\rangle$

- intransitive verbs (*exhales*)
- intransitive nouns (*goalkeeper*)
- intransitive adjectives (*smart*)
- intransitive prepositions (*off*)

2.2 4 sentences, the same compositional structure

(13) Celia exhales.

(14) Hope is a goalkeeper.

(15) Aïda is smart.

(16) Dobby is off.

We ignore copulas and articles and tense and agreement … for now.

2.3 Adding two-place/transitive predicates

(17) Liza knows Greek.

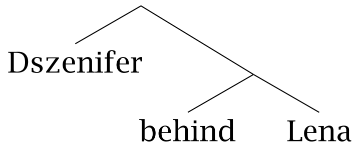
(18) Abby is a student of combinatorics.

(19) Jennifer is proud of Linda.

(20) Dzsenifer is behind Lena.

2.4 The tree for a transitive sentence

(21)



2.5 Function-valued functions

(22) For any world w,
$[\![\text{behind}]\!]^w = \lambda x\colon\ x \in D_e.\,(\lambda y\colon\ y \in D_e.\ y \text{ is behind } x \text{ in } w)$

In prose: “the extension of *behind* relative to a world w is the function that maps any individual x to the function that maps any individual y to the truth-value 1 if and only if y is behind x in w”.

Because natural language has the binary branching structure created by MERGE, the semantics works best with the curried/schönfinkeled method of feeding arguments to functions one by one, with functions as the intermediate output along the way.

3 Negation

Here’s a sentence from the Gude language spoken in Nigeria (Hoskison 1983: p. 80):

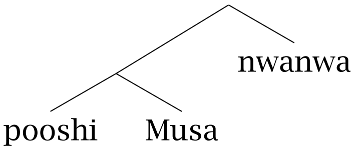
(23)

pooshi	Musa	nwanwa
NEGATIVE	Musa	chief

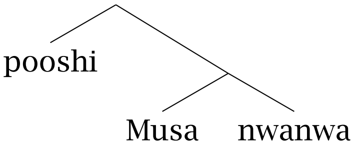
‘Musa is not a chief’

There are two initially possible tree structures:

(24) a.



b.



We take for granted that *Musa* is of type e, that *nwanwa* is of type $\langle e, t\rangle$, and that the entire sentence (like all declarative sentences is of type t, that is: relative to a world, the sentence denotes a truth-value).

Given that we assume until forced otherwise that semantic composition occurs via Function Application, this means that we have the following options:

1. Tree (24a) and a meaning for *pooshi* of type $\langle e, e\rangle$
2. Tree (24a) and a meaning for *pooshi* of type $\langle e, \langle\langle e, t\rangle, t\rangle\rangle$
3. Tree (24b) and a meaning for *pooshi* of type $\langle t, t\rangle$

We considered the first possibility in class on Monday and reasoned that *pooshi* would then be taking an individual as its argument and returning an individual that (in the relevant world) has all and only the properties that the original doesn't have. So, if "not-Musa" is chief, then that's the same as Musa not being chief. The idea would be that for every individual there is an anti-individual. But there are two problems: (i) do we really think that Gude speakers (or we for that matter) think that for the sentence to be true we need a "not-Musa" individual to exist? and (ii) take a property that is true of any individual ("is self-identical"); then the sentence "pooshi Musa self-identical" is surely false (since it's claiming that Musa is not self-identical and all individuals are self-identical); at the same time, "not-Musa" is an individual and thus self-identical and so, the sentence "pooshi Musa self-identical" should be true. Contradiction!

The other two options are feasible. Writing a meaning for *pooshi* under Option 2 is left as an exercise.

We assume that Option 3 is most likely correct. *pooshi* is a function takes a truth-value and returns the "opposite" truth-value. If its argument sentence is true, then the *pooshi*-sentence is false, and vice versa. So, *pooshi Musa nwanwa* says that the sentence *Musa nwanwa* is false.

(25) For any world w,
$[\![\text{pooshi}]\!]^w = \lambda u_t.\ u = 0$

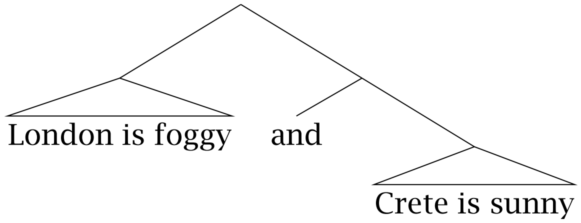
4 Conjunction

Consider the sentence

(26) London is foggy and Crete is sunny.

We assume the following binary branching structure (with the internal makeup of the two conjoined sentence hidden via the triangle roof convention):

(27)



We deduce that *and* has to be of type $\langle t, \langle t, t\rangle\rangle$ and propose the following meaning:

(28) For any world w,
$[\![\text{and}]\!]^w = \lambda u_t.\,(\lambda v_t.\, u = 1 \text{ and } v = 1)$

In class, we discussed that the conjunction *and* can appear in other uses as well:

(29) a. London is foggy and rainy. □pset01

b. London and Crete are sunny.

We won't have time to figure out how (29b) might work.

5 Modification

The intransitive predicate *foggy* can be used to attribute a property of an individual (as in *London (is) foggy*). But it can also be used to modify another predicate (as in *London (is a) foggy town*). How does this work?

Options:

1. One of the "predicates" is of type $\langle et, et\rangle$
2. A special composition principle applies

We'll consider these options next week. To prepare, you can optionally read the relevant discussion from Portner (2005: Sections 4.1-4.2), posted on Canvas.

References

Heim, Irene & Angelika Kratzer. 1998. *Semantics in generative grammar*. Oxford: Blackwell.

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