24.903 Week #8 – 2022-03-28 + 2022-03-30

Kai von Fintel

1 Quantifiers

We will be looking at the meaning of expressions like the following:

$$\left\{\begin{array}{l}\text{every}\\ \text{some}\\ \text{no}\\ \text{exactly two}\\ \text{at least two}\\ \text{at most two}\\ \text{both}\\ \text{neither}\\ \text{more than half (of)}\\ \text{most}\\ \text{many}\\ \text{several}\\ \text{few}\\ \text{a few}\\ \text{all but five (of)}\\ \text{the four}\\ \text{between five and ten}\\ \text{an even number of}\end{array}\right\} \text{(the) child(ren)} \left\{\begin{array}{l}\text{is}\\ \text{are}\end{array}\right\} \text{tall}$$

We discussed four conceivable approaches to the analysis of *most*, depending on both syntactic constituency and semantic plausibility. We settled on the following: *most* first combines with a nominal predicate (*squares*, for example) and then combines with the verb phrase predicate:

(1) [most squares] are filled.

Both of the arguments of *most* are of type $\langle e,t\rangle$ and thus *most* is of type $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$. In other words, *most* denotes a relation between two sets. A concrete proposal for its meaning is:

(2) For any context c and world w:
$[\![\text{most}]\!]^{c,w} = \lambda f_{e,t}.(\lambda g_{e,t}.\ |char_f \cap char_g| > |char_f - char_g|)$

where for any function f of type $\langle e,t\rangle$, $char_f = \{x \in D_e\colon\ f(x) = 1\}$, the set characterized by f.

We can now specify the meaning for other quantifiers schematically, by saying what they require of two sets A and B that they are given as arguments. For example:

$$\begin{aligned}
\text{every}(A)(B)&\colon\ A \subseteq B\\
\text{some}(A)(B)&\colon\ A \cap B \neq \emptyset\\
\text{exactly three}(A)(B)&\colon\ |A \cap B| = 3\\
\text{no}(A)(B)&\colon\ A \cap B = \emptyset
\end{aligned}$$

2 Conceivable relations between two sets

There's a large set of conceivable meanings for quantifiers that are not instantiated in any language. Here's a sample:

$$\begin{aligned}
\text{Dsz}(A)(B)&\colon\ \text{Dszenifer} \in A \cap B\\
\text{Allam}(A)(B)&\colon\ \forall x \in A\ \exists y \in B \text{ such that } x \text{ admires } y \text{ in w}\\
\text{Moxon}(A)(B)&\colon\ |A| > |D_e - B|\\
\text{Evso}(A)(B)&\colon\ A \subseteq B, \text{ if } |D_e| < 1{,}000{,}000\\
&\ \ A \cap B \neq \emptyset, \text{ otherwise}\\
\text{AllbutJohn}(A)(B)&\colon\ A - j \subseteq B\\
\text{Somesquare}(A)(B)&\colon\ \text{there is a square in } A \cap B\\
\text{Mmore}(A)(B)&\colon\ |A| > |B|\\
\text{Equi}(A)(B)&\colon\ |A| = |B|\\
\text{Three*}(A)(B)&\colon\ |A| \geq 3\\
\text{Some-not}(A)(B)&\colon\ A - B \neq \emptyset
\end{aligned}$$

Intuitions: natural language quantifiers

- care only about A and B, nothing else
- they "live on" A, A "sets the scene", A is the domain

- they care about "logical" (numerical?) relations

We will define three formal properties of relations between sets that are meant to capture these intuitions.

3 Three constraints on quantifier meanings

3.1 EXTENSION

The EXTENSION constraint requires that there be no sensitivity to any elements of D_e outside of A and B. Technically, this is formulated as the condition that as long as A and B are both subsets of D_e, it doesn't matter if we add further elements to D_e.

The intuition:

- only the relation between two sets characterized by the predicates matters
- if the set of entities (D_e) were larger (or smaller), it wouldn't matter, as long as the two sets stay the same
- in effect, as far as natural language quantifiers are concerned, we might as well have a D_e that is identical to $A \cup B$

The constraint rules out the putative quantifiers *moxon* and *evso* defined earlier.

3.2 ISOMORPHY

The intuition:

- the particular identity and nature of the entities in A and B doesn't matter
- what matters are "structural" relations between the sets

More formally:

(3) A permutation of a set is a bijective function from the set to the set again. Each element of the set is mapped to a unique, possibly different element of the set.

(4) A quantifier Q satisfies ISOMORPHY iff
$\forall$ permutations π of D_e: $Q(A)(B) = Q(\pi(A))(\pi(B))$

Which quantifiers from our list are ruled out by ISOMORPHY?

3.3 CONSERVATIVITY

(5) A quantifier Q is CONSERVATIVE iff
$\forall A, B$: $Q(A)(B) = Q(A)(A \cap B)$

The intuition: quantifiers do not care about what happens outside A.

- The first argument of a quantifier "sets the scene".
- A quantifier "lives on" its first argument.

A test for CONSERVATIVITY:

- Every box is blue = every box is a box that is blue.
- Some box is blue = some box is a box that is blue.
- No box is blue = no box is a box that is blue.
- Three boxes are blue = three boxes are boxes that are blue.
- Most boxes are blue = most boxes are boxes that are blue.
- Few boxes are blue = few boxes are boxes that are blue.
- Many boxes are blue = many boxes are boxes that are blue.

Now ruled out by CONSERVATIVITY: *mmore*, *equi*

4 Number trees

All three constraints together result in quantifier meanings that can be stated as conditions on just two numbers:

- $|A \cap B|$: the number of elements in $A \cap B$ and
- $|A - B|$: the number of elements in $A - B$

This has a wonderfully simple visualization.

0,0

1,0 0,1

2,0 1,1 0,2

3,0 2,1 1,2 0,3

4,0 3,1 2,2 1,3 0,4

5,0 4,1 3,2 2,3 1,4 0,5

Pick a quantifier, any quantifier, and highlight the number pairs that verify the quantifier.

5 More formal properties of quantifiers

While the three formal properties of natural language quantifiers that we have discussed so far are arguably universal, there are others that determine (at least mathematically, but perhaps also linguistically) natural classes of quantifiers.

Any quantifier Q is

reflexive	$\forall A\colon\ Q(A)(A)$
irreflexive	$\forall A\colon\ \neg Q(A)(A)$
symmetric	$\forall A, B\colon\ Q(A)(B)$ iff $Q(B)(A)$
anti-symmetric	$\forall A, B\colon$ if $Q(A)(B)$ & $Q(B)(A)$, then $A = B$
transitive	$\forall A, B, C\colon$ if $Q(A)(B)$ & $Q(B)(C)$, then $Q(A)(C)$
left upward monotone	$\forall A, B, C\colon$ if $A \subseteq B)$ & $Q(A)(C)$, then $Q(B)(C)$
left downward monotone	$\forall A, B, C\colon$ if $A \subseteq B)$ & $Q(B)(C)$, then $Q(A)(C)$
right upward monotone	$\forall A, B, C\colon$ if $A \subseteq B)$ & $Q(C)(A)$, then $Q(C)(B)$
right downward monotone	$\forall A, B, C\colon$ if $A \subseteq B)$ & $Q(C)(B)$, then $Q(C)(A)$

In class: in each case, think of quantifiers that have the property and ones that don't.