Linear Algebra and Optimization
MIT Fall 2022

Recitation 17

Tuesday November 8, 2022

Directions Go over problems from Recitation 16 if any were not covered before moving to this recitation.

1 Recap

1.1 Symmetric Matrices

Symmetric matrices are important to study, often appearing in applications such as the Hessian second derivative and optimization problems. A matrix $A \in \mathbb{R}^{n \times n}$ is symmetric if $A = A^\top$.
With real symmetric matrices, all eigenvalues λ_i are real and eigenvectors can always be chosen to be orthogonal. We can then create the eigendecomposition

$$A = TDT^\top \qquad \text{or equivalently} \qquad A = \sum_{i=1}^{n} \lambda_i v_i v_i^\top$$

where A is a real symmetric matrix, T is the matrix of orthogonal eigenvectors, and D is a diagonal matrix of eigenvalues.

1.2 Quadratic Functions as Matrices

Oftentimes, we want to express quadratic functions as matrices to solve in an optimization problem. Any quadratic function $f(x_1, ..., x_n)$ can be expressed as

$$x^\top Ax + b^\top x$$

where A is a symmetric matrix $\in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$ are coefficient matrices, and $x = [x_1 \ ... \ x_n]^\top$ represents a vector of n variables.

2 Exercises

Initially go over exercises from previous recitation not covered

1. True or False
   (a) If a matrix A is symmetric and invertible, so is A^{-1}.
   (b) For any matrix A, the matrix AA^T is symmetric.
2. Write the following equations in matrix-vector form $x^TAx + b^Tx$
   (a) $4x^2 - 6xy + 2y^2 + 7x - 35y$
   (b) $\frac{5}{2}x^2 - 2xy - xz + 2y^2 + 3yz + \frac{5}{2}z^2 + 2x - 35y - 47z$
3. Solve the following linear ODE. Use Julia to calculate any inverses of matrices.

$$\frac{dx(t)}{dt} = -6x(t) + 3y(t)$$

$$\frac{dy(t)}{dt} = 4x(t) + 5y(t)$$