

Axioms. All instances of the following schemata. v_i, v_j , and v_k can be any of the variables v_1, v_2, v_3, \dots c and d are constants.

- (K) $\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$
- (\Diamond Def) $\Diamond\phi \leftrightarrow \sim\Box\sim\phi$
- (\forall Dist) $(\forall v_i)(\phi \rightarrow \psi) \supset ((\forall v_i)\phi \supset (\forall v_i)\psi)$
- (RUS) $(\forall v_i)\phi(v_i) \rightarrow ((\exists v_k)v_k = c \rightarrow \phi(c))$
- (Vac) $\phi \leftrightarrow (\forall v_i)\phi$, v_i not free in ϕ
- (EE) $(\forall v_i)(\exists v_j)v_i = v_j$
- (\exists Def) $(\exists v_i)\phi \leftrightarrow \sim(\forall v_i)\sim\phi$.
- (Ref=) $c = c$
- (Sub=) $c = d \rightarrow (\phi(c) \leftrightarrow \phi(d))$
- ($\Box \neq$) $\sim c = d \rightarrow \Box\sim c = d$

Rules.

TC. You may write any tautology or any tautological consequence of things you've written earlier.

Nec. From ϕ , you may infer $\Box\phi$.

(UG) From $\phi(c)$, you may infer $(\forall v_i)\phi(v_i)$, provided c doesn't appear in $\phi(v_i)$.

(UGⁿ) From $(\psi_1 \rightarrow \Box(\psi_2 \rightarrow \Box(\psi_3 \rightarrow \dots \Box(\psi_{n-1} \rightarrow \Box\psi_n(c))\dots)))$, you may infer $(\psi_1 \rightarrow \Box(\psi_2 \rightarrow \Box(\psi_3 \rightarrow \dots \Box(\psi_{n-1} \rightarrow \Box(\forall v_i)\psi_n(v_i))\dots)))$, provided c doesn't appear in any of the ψ_k s with $1 \leq k < n$ or in $\psi_n(v_i)$, and v_i occurs free in $\psi_n(v_i)$.