Free Logic

Presupposition-free logic (*free logic* for short) is predicate logic freed of the presupposition that individual constants always denote. Our models of free logic will consist of an ordinary model $\mathfrak{A}$ together with a nonempty subset D of $|\mathfrak{A}|$, called the *actual* or *inner domain*. D is intended domain of quantification. $|\mathfrak{A}| \sim D$, the *fictitious* or *outer domain* consists of individuals from outside the intended domain of quantification that are enlisted to play the roles of fictitious entities. We shouldn't think of $|\mathfrak{A}| \sim D$ as consisting of nonexistent things, since presumably there are no nonexistent things.

A sentence φ is true in the free model $<\mathfrak{A},D>$ iff φ^E is true in the ordinary model we get from $\mathfrak{A}$ by adding a new predicate "E" whose extension is D. Here φ^E is the sentence we get from φ by restricting the quantifiers to "E," replacing $(\forall v_i)$ with $(\forall v_i)(E(v_i) \rightarrow ...)$ and replacing $(\exists v_i)$ with $(\exists v_i)(E(v_i) \wedge ...)$.

Assuming the intended domain of quantification is infinite, we could do the same things with a single domain by letting the members of the domain play two roles. In addition to its day job, acting as a member of a model in the usual way, a member of the domain might have a second job acting as a character in a fiction. This would be a little more complicated, but it would let us obey Father Parmenides' advice: "Things that are not in no way are, but keep your mind from that path of inquiry." (Quoted by Plato in the *Sophist* 237a.)

Axioms of free logic:

($\forall$Dist)	$(\forall v_i)(\varphi \rightarrow \psi) \rightarrow ((\forall v_i)\varphi \rightarrow (\forall v_i)\psi)$
(RUS)	$(\forall v_i)\varphi(v_i) \rightarrow (\exists v_j)(v_j = c \rightarrow \varphi(c)))$
(Vac)	$(\varphi \leftrightarrow (\forall v_i)\varphi)$, v_i not free in φ
($\exists$Def)	$(\exists v_i)\varphi(v_i) \leftrightarrow \sim (\forall v_i) \sim \varphi(v_i)$
(EE)	$(\forall v_i)(\exists v_j)v_i = v_j$
(Ref=)	$c = c$
(Sub=)	$(c=d \rightarrow (\varphi(c) \leftrightarrow \varphi(d))$

Rules:

TC	
UG	If c doesn't occur in $\varphi(v_i)$, then from $\varphi(c)$ you may infer $(\forall v_i)\varphi(v_i)$.

The proofs of soundness and completeness are almost unchanged. In forming Γ_{n+1} in the case where $\Gamma_n \cup \{\xi_n\} \nvdash \chi$ and ξ_n has the form $(\exists v_j)\psi(v_j)$, let Γ_{n+1} be $\Gamma_n \cup \{\xi_n, \psi(c_i), (\exists v_j)v_j = c_i\}$. When we form the model, $\mathfrak{A}(c_i)$ will be in the inner domain D iff $(\exists v_j)v_j = c_i)$ is in Γ_∞.