The Garden of Forking Paths[1]

Some events happen by law, and others by chance. An event happens by law if, given the laws of nature, that it would happen was ensured by thing that happened previously. Sometimes, however, there is an actual or metaphorical roll of the dice, a chance even with several possible outcomes, each of which initiates a possible future history. We can think of possible histories of the world – past, present, and future – as paths through a branching tree. If two possible histories agree in their accounts of everything that happened up till time t, when some chance event sent them in different directions, than we'll say the histories coincide up to t. If every history of the world that agrees with the actual history up to the present time is one according to which there is a sea battle tomorrow, then it is determined that there will be a sea battle tomorrow. If instead it is a matter of chance whether there will be a sea battle tomorrow, then there will be some histories that coincide with the actual history up to the present time in which there is a sea battle tomorrow and others in which there isn't one. It is determined that either there will be a sea battle or there will not, but it isn't either determined that there will be a sea battle or determined that there will not be one. One of the other of the following statements is now true:

> There will be a sea battle tomorrow but it isn't determined that there will be a sea battle tomorrow.
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> There will be no sea battle tomorrow, but it isn't determined that there will be no sea battle tomorrow.

It is not now determined which of the two is true. (The example is from Aristotle, *De Interpretatione*, ch. 9.)

The past and the present are settled, but the future may well be open. Thus what is determined changes from time to time. Yesterday, it wasn't determined what town I would be in at 2:15 today, but now, at 2:00, it's determined that at 2:15 I will be in Brookline.

We want to redeploy our formalism, interpreting "□" as "It is determined that," rather than "It is necessary that." Some of the formal laws aren't changed very much. If it's determined that φ is true, φ is true. So (T) holds. If it's determined that (φ → ψ) is true and also that φ is true, then it's determined that ψ is true. So we have (K). Tautologics hold no matter what, so we have (Taut).

Let's look at (4). Suppose it is now determined that φ, so that □ φ is true now. This means that φ is true in any possible history that agrees with the actual history up till now. Let h be a history that agrees with the actual history up till now and let t be a time in the future. Every history h* that agrees with h up till t will agree with the actual history up till now, so φ will be true in h*. Since φ is true in every history that agrees with h up till t, it will be determined in h at t that φ is true; that is, □ φ will be true in h at t. We have shown that □ φ will be true in every

[1]"The Garden of Forking Paths" is the title of a story by Jorge Luis Borges, in his collecton *Labyrinths* (New Directions: 1962).

history that agrees with the actual history up till now, so it is now determined that $\Box\ \varphi$ is true, and $\Box\ \Box\ \varphi$ is true.

(5) is a different. It, let us assume, compatible with the laws of nature and the history of the world up till now that I should walk to Cambridge tomorrow. It is also possible, although extremely unlikely,[2] that tonight I should be struck by a meteor and killed. If I am killed tonight by a meteor, that I should l walk to Cambridge tomorrow will no longer be a possibility. That I should walk to Cambridge tomorrow is possible, but it is possible for events to unfold in such a way that it will no longer be possible for me to walk to Cambridge tomorrow. It is possible that I should walk to Cambridge tomorrow, but it isn't determined that it is possible that I should walk to Cambridge tomorrow. So ($\Diamond$ I walk to Cambridge tomorrow $\rightarrow$ $\Box\ \Diamond$ I walk to Cambridge tomorrow), an instance of (5), is untrue. (In putting things this way, I am assuming that I won't be killed by a meteor tonight, although I can't possible now know this for sure.)

In the modal sentential calculus, as in the plain sentential calculus, it's not our business to try to understand when and why the atomic sentences are true. Let's assume that the truth conditions of the atomic sentences are already set, and see how to apply the formalism to compound sentences. Let W be the set of all pairs <h,t>, where h is a complete story of the world and t is an instant in time. To say it's a complete history means that, for each sentence φ, either φ or ~ φ is true in h. Our interpretation function I assigns the value True to <α,<h,t>> iff the atomic sentence α is true in h. The new notion we need to apply the formalism is the idea of one pair in W having access to another. <h*,t*> is accessible from <h,t> if t* is the same at t or later than t and, as far as things are determined by h up to time t, h* might be true. That is, writing "R" for "has access to," <h,t> R <h*,t*> iff t* is later than or identical to t and h* and h coincide up till time t. To say that $\Box\ \varphi$ is true at <h,t> means that φ is true in every complete history that coincides with h up till time t. In other words, $\Box\ \varphi$ is true in <h,t> iff φ is true in every <h*,t*> with <h,t> R <h*,t*>.

R is reflexive, since h coincides with itself up till time t. R is also transitive. If <h,t> R <h*,t*> and <h*,t*> R <h#,t#>, then t is earlier than or equal to t* and t* is earlier than or equal to t#, so t is earlier than or equal to t#. h and h* coincide up till time t. h* and h# coincide up till time t*, so they coincide up till time t. Since h and h* coincide up till time t and h* and h# coincide up till time t, h and h# coincide up till time t. So <h,t> R <h#,t#>.

An atomic sentence α is true in <h,t> iff <α,<h,t>> is assigned True by I.

A disjunction is true in <h,t> iff one or both disjuncts are true in <h,t>.

A conjunction is true in <h,t> iff both conjuncts are true in <h,t>.

A negations is true in <h,t> iff the negatum isn't true in <h,t>.

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[2] I am told that there is only one documented case of a person being struck by a meteor. She wasn't badly injured.

□ φ is true in <h,t> iff φ is true in every pair <h*,t*> with <h,t> R <h*,t*>,

We have seen that every formula of any of the forms (Taut), (K), (T) and (4) is true at every pair <h,t>. Moreover, if a conditional and its antecedent are true at every pair in W, the consequent is true at every pair in W. If φ is true in every pair in W, □ φ is true in every world in W. That is, the set of sentences true in every pair in W is closed under (MP) and (Nec). Lewis's system S4 is the smallest class of sentences that contains (Taut), (K), (T), and (4) and is closed under (MP) and (Nec). Every sentence in S4 is true at every member of W.