

Subject 24.244. Modal Logic. Problem set due Tuesday, September 29

1. KT5 is the smallest normal modal system that contains (T) and (5).
 - a) Show, by giving a derivation, that every instance of schema (B) is in KT5.
 - b) Show, by giving a derivation, that every instance of schema (4) is in KT5.
 - c) Show that any binary relation that is Euclidean is transitive and symmetric.
2. KB4 is the smallest normal modal system that contains (B) and (4)
 - a) Show, by giving a derivation, that every instance of schema (5) is in KB.
 - b) Show that any binary relation that is symmetric and transitive is Euclidean.
3. K45 is the smallest normal modal system that contains (4) and (5).
 - a) Show that a modal system can be transitive and Euclidean without being symmetric.
 - b) Show that $(P \rightarrow \Box \Diamond P)$ isn't in K45.
4. Show that every modal formula can be derived in the system obtained from Strict S5 by incorporating Substitution as an additional rule, in addition to (MP) and (Nec).
5. Find English sentences that you can substitute for "P" and "Q" so that, when " $\Diamond$ " is translated as "it is logically possible that," the English translation of $(P \wedge Q) \wedge \sim \Diamond(P \wedge \sim Q) \wedge \sim \Diamond(\sim P \wedge Q) \wedge \Diamond(\sim P \wedge \sim Q)$ comes out true. Explain your answer.
6. Find English sentences that you can substitute for "P" and "Q" so that, when " $\Diamond$ " is translated as "it is logically possible that," the English translation of $\Diamond(P \wedge Q) \wedge \Diamond(P \wedge \sim Q) \wedge \sim \Diamond(\sim P \wedge Q) \wedge (\sim P \wedge \sim Q)$ comes out true. Explain your answer.
7. Let's say that two sentences ϕ and ψ are Strict-S5-equivalent iff $(\phi \leftrightarrow \psi)$ is in Strict S5. How many nonempty Strict-S5-equivalence classes of modal formulas containing no atomic formulas other than "P," "Q," and "R" are there? You don't need to list them. Just say how many there are. Explain your answer.
8. Let's say that two sentences ϕ and ψ are S5-equivalent iff $(\phi \leftrightarrow \psi)$ is in S5. How many nonempty S5-equivalence classes of modal formulas containing no atomic formulas other than "P," "Q," and "R" are there? Explain your answer.