Subject 24.244. Modal Logic. Problem set due Tuesday, Sept. 22.

1. Consider SC connectives "NAND" and "XOR," with the following truth tables:

φ	ψ	(φ NAND ψ)	(φ XOR ψ)
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	F

a) Given a sentence containing only the connective "NAND" that is logically equivalent to "(P XOR Q," or explain why there can be no such connective.

b) Given a sentence containing only the connective "XOR" that is logically equivalent to "(P NAND Q," or explain why there can be no such connective.

2. Use the Compactness Theorem to show that, for Γ and Δ sets of SC sentences, the following are equivalent:

   $\Gamma \cup \Delta$ is inconsistent.

   There is a sentence φ such that Γ implies φ and Δ implies $\sim \varphi$.

3. Within the version of the sentential calculus in which the atomic sentences are uppercase letters from the English alphabet, with or without Arabic numeral subscripts, let us say that a set **S** of complete stories is *closed* iff there is a set of sentences Γ such that **S** = {complete stories that include Γ}.

a) True or false? Explain your answer: The intersection of two closed sets of complete stories is always closed.

b) True or false? Explain your answer: The union of two closed sets of complete stories is always closed.

c) Let's say a set of complete stories is *clopen* if it and its complement are both closed. Show that a set of complete stories is clopen iff there is a sentence φ with **S** = {complete stories that include φ}.

d) True or false? Explain your answer: The complement of a closed set of complete stories is always closed.

4. Would any of the answers to problem 3 have changed if we were talking about the language whose atomic sentences are the 26 uppercase English letters, without the numerical subscripts? Explain your answer.

5. For each of the following sentences, either give a derivation in S5 or present a simple Kripke model in which it's false. In doing the derivations, you may use the derived rules from the lecture notes.

a) $((\Box P \vee \Box Q) \equiv \Box(P \vee Q))$

b) $((\Box P \wedge \Box Q) \equiv \Box(P \wedge Q))$

c) $((\Diamond(P \supset Q) \supset (\Diamond P \supset \Diamond Q))$

d) $((\Diamond P \supset \Diamond Q) \supset \Diamond(P \supset Q))$

e) $(\Box(P \vee (\Diamond Q \vee \Box R)) \equiv (\Box P \vee(\Diamond Q \vee \Box R)))$

f) $(\Box \Diamond \Box (P \equiv Q) \equiv \Diamond \Box \Diamond (P \equiv Q))$