

Subject 24.244. Modal Logic. Problem Set due Tuesday, October 6

1. Which of the following formulas are in S4? If the formula is in S4, give a derivation. If it isn't in S4, give a transitive, reflexive model in which it is false.
 - a) $(\Box P \supset \Box \Diamond \Box P)$
 - b) $(\Diamond P \supset \Diamond \Box \Diamond P)$
 - c) $(\Box(P \vee \Box Q) \equiv (\Box P \vee \Box \Box Q))$
2. True or false? Explain your answer: The following are equivalent, for any formula ϕ :
 - a) ϕ is true in every model $\langle W, R, I, @ \rangle$ with R a reflexive, transitive, symmetric relation on W.
 - b) ϕ is true in every model $\langle W, R, I, @ \rangle$ with R equal to $W \times W$.
3. A binary relation R on a set W is a partial function iff, whenever Rwu and Rwv , we have $u=v$. Give a system of axioms that generates all the formulas true in every model $\langle W, R, I, @ \rangle$ in which R is a partial function, and show that it works.
4. A binary relation R on a set W is a total function iff it's a partial function whose domain is all of W. Give a system of axioms that generates all the formulas true in every model $\langle W, R, I, @ \rangle$ in which R is a total function, and show that it works.
5. Show that there is an algorithm for testing whether a formula is true in every model $\langle W, R, I, @ \rangle$ in which R is a total function.
6. Which of the systems KT, K4, KT4, and KT5 contain the formula " $((\Diamond P \wedge \Diamond Q) \leftrightarrow \Diamond(\Diamond P \wedge \Diamond Q))$ "? Explain your answers.
7. True or false? Explain your answer: A disjunction $(\Box \phi \vee \Box \psi)$ is in S4 if and only if either $\Box \phi$ is in S4 or $\Box \psi$ is in S4.
8. True or false? Explain your answer: A disjunction $(\Box \phi \vee \Box \psi)$ is in S5 if and only if either $\Box \phi$ is in S5 or $\Box \psi$ is in S5.