

Rules of Intuitionistic Natural Deduction

\vdash_{Int} is the smallest relation relating sets of sentences to sentences that meets the following conditions:

<i>Identity</i>	If $\gamma \in \Gamma$, then $\Gamma \vdash_{\text{Int}} \gamma$.
<i>Transitivity</i>	If $\Gamma \vdash_{\text{Int}} \delta$ for each member δ of Δ and $\Delta \vdash_{\text{Int}} \phi$, then $\Gamma \vdash_{\text{Int}} \phi$.
<i>“\wedge”-introduction</i>	$\{\phi, \psi\} \vdash_{\text{Int}} (\phi \wedge \psi)$.
<i>“\wedge”-elimination</i>	$\{(\phi \wedge \psi)\} \vdash_{\text{Int}} \phi$ and $\{(\phi \wedge \psi)\} \vdash_{\text{Int}} \psi$.
<i>“\vee”-introduction</i>	$\{\phi\} \vdash_{\text{Int}} (\phi \vee \psi)$, and $\{\psi\} \vdash_{\text{Int}} (\phi \vee \psi)$.
<i>Proof by cases</i>	If $\Gamma \cup \{\phi\} \vdash_{\text{Int}} \theta$ and $\Gamma \cup \{\psi\} \vdash_{\text{Int}} \theta$, then $\Gamma \cup \{(\phi \vee \psi)\} \vdash_{\text{Int}} \theta$.
<i>Modus ponens</i>	$\{\phi (\phi \rightarrow \psi)\} \vdash_{\text{Int}} \psi$.
<i>Conditional proof</i>	If $\Gamma \cup \{\phi\} \vdash_{\text{Int}} \psi$, $\Gamma \vdash_{\text{Int}} (\phi \rightarrow \psi)$.
<i>Ex contradictione quodlibet</i>	$\{\perp\} \vdash_{\text{Int}} \chi$, any χ .
<i>Law of contradiction</i>	$\{\phi, \sim\phi\} \vdash_{\text{Int}} \perp$.
<i>Intuitionistic reductio</i>	If $\Gamma \cup \{\phi\} \vdash_{\text{Int}} \perp$, $\Gamma \vdash_{\text{Int}} \sim\phi$.

To get classical logic, take \vdash_{Class} to be the smallest relation that meets the eleven conditions above, together with:

Double negation elimination $\{\sim\sim\phi\} \vdash_{\text{Class}} \phi$.