An atomic formula is bounded
If φ and ψ are bounded, so are $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$, and $\sim \varphi$.
If φ is bounded, so are $(\forall x \leq \tau)\varphi$ and $(\exists x \leq \tau)\varphi$, where "x" doesn't occur within τ.

A Σ formula is obtained by prefixing existential quantifiers to a bounded formula.
A set or relation is Σ iff it's the extension of a Σ formula.
A set of relation is Δ iff it and its complement are Σ.

Church-Turing Thesis. A partial function is calculable iff it's the extension of a Σ formula.

A set or relation is effectively enumberable iff it's Σ.

A set or relation is decidable iff it's Δ.

Every true Σ sentence is provable in PA.

Every effectively enumerable set is *weakly representable* in PA: There is a formula φ such that, for every n, n is in the set iff PA $\vdash \varphi[n]$.

Every decidable set S is *strongly representable* in PA: There is a formula ψ such that, for any in, if $n \in S$, $\psi([n]) \in$ PA, and if $n \notin S$, PA $\vdash \sim \psi([n])$.

Every calculable total function f is *functionally representable* in PA: There is a formula $\mathscr{F}$ so that, for any n, PA $\vdash (\forall y)(\mathscr{F}([n],y) \leftrightarrow y = [f(n)])$.