

Subject 24.244. Modal logic. Problem set due Thursday, Oct. 22

1. Describe a frame  $\langle W, R, I \rangle$  with the properties that every formula in KT is true in every world in the frame and that R isn't reflexive, and explain why it has those properties; or else, explain why there is no such frame.
2. Describe a frame  $\langle W, R, I \rangle$  with the properties that every formula in K4 is true in every world in the frame and that R isn't transitive, and explain why it has those properties; or else, explain why there is no such frame.
3. Which of the following formulas are in GL? Explain your answers:
  - a)  $((\Diamond P \wedge \Diamond Q) \rightarrow \Diamond(\Diamond P \wedge \Diamond Q))$ .
  - b)  $(\Diamond(\Diamond P \wedge \Diamond Q) \rightarrow (\Diamond P \wedge \Diamond Q))$ .
  - c)  $(\Diamond(P \leftrightarrow Q) \rightarrow \Box\Diamond(P \leftrightarrow Q))$ .
  - d)  $(\Diamond(\Diamond P \wedge \Diamond Q) \leftrightarrow (\Diamond\Diamond P \wedge \Diamond\Diamond Q))$ .
4. Show that, when we defined GL, including axiom schema (4) was redundant, so that GL is equal to KL, the smallest normal modal system that contains (L); this was discovered by Dick de Jongh. [Hint: The relevant instance of (L) is  $(\Box(\Box(\varphi \wedge \Box \varphi) \rightarrow (\varphi \wedge \Box \varphi)) \rightarrow \Box(\varphi \wedge \Box \varphi))$ .]
5. Take a sentence  $\alpha$  so that  $\alpha$  is provably (in PA) equivalent to  $\neg \text{Bew}_{\text{PA}}([\ulcorner \neg \alpha \urcorner])$ . Is  $\alpha$  decidable in PA? Is it true (in the standard model)? Explain your answers..
6. Take a sentence  $\beta$  so that  $\beta$  is provably equivalent to  $(\text{Bew}_{\text{PA}}([\ulcorner \beta \urcorner]) \vee \sim \text{Bew}_{\text{PA}}([\ulcorner \beta \urcorner]))$ . Is  $\beta$  decidable in PA? Is it true? Explain your answer.
7. Take a sentence  $\delta$  so that  $\delta$  is provably equivalent to  $(\text{Bew}_{\text{PA}}([\ulcorner \delta \urcorner]) \wedge \text{Bew}_{\text{PA}}([\ulcorner \neg \delta \urcorner]))$ . Is  $\delta$  decidable in PA? Is it true?
8. Show that every sentence of the form  $\Diamond \Box \varphi$  is in GLS.