

The following axioms use the operators “G” to represent “it’s always going to be the case that,” “F” to represent “it will sometime be the case that,” “H” to represent “It has always been the case that,” and “P” to represent “it was sometimes the case that.” They assume that time is totally ordered, dense, and continuous, without beginning or end:

Axioms. We use “Pp” to abbreviate “ $\sim H\sim p$ ” and “Fp” to abbreviate “ $\sim G\sim p$.”

- (i) $(G(p \rightarrow q) \rightarrow (Gp \rightarrow Gq))$
- (ii) $(p \rightarrow GPp)$
- (iii) $(Gp \rightarrow GGp)$
- (iv) $((Fp \wedge Fq) \rightarrow (F(p \wedge Fp) \vee (F(p \wedge q) \vee F(Fp \vee q))))$
- (v) $(Gp \rightarrow Fp)$
- (vi) $(Fp \rightarrow FFp)$
- (vii) $((Fp \wedge FG\sim p) \rightarrow F(HFp \wedge G\sim p))$

Rules.

Tautological consequence.

Temporal generalization: From ϕ to infer $G\phi$ and $H\phi$.

Mirror image rule: From a formula, you may derive the corresponding formula obtained by exchanging “G” and “H” and also “F” and “P.”