Subject 24.244. Modal Logic Problem set due Thursday, October 29.

Hamblin's 15 tense theorem tells us about sequences of tense operators starting with "H" ("it has always been the case that"), "P" ("it has sometimes been the case that"), "G" ("it will always be the case tbat"), and "F" ("it will sometimes be the case that"). He gave a diagram of 15 prefixes (including the null prefix). He showed that one of the 15 prefixes implies another if and only is they are connected by a path of arrows, and that any prefix optained by prefixing one of the operators to a prefix in the diagram is eqivalent to a prefix in the diagram. Here, to say that φ implies ψ means that $(\varphi \rightarrow \psi)$ is derivable within the following system of axioms, which assume that time is totally ordered, dense, and continuous, without beginning or end.

Axioms. We use "Pp" to abbreviate "~H~p" and "Fp" to abbreviate "~G~p."

(i) $(G(p \rightarrow q) \rightarrow (Gp \rightarrow Gq))$
(ii) $(p \rightarrow GPp)$
(iii) $(Gp \rightarrow GGp)$
(iv) $((Fp \wedge Fq) \rightarrow (F(p \wedge Fp) \vee (F(p \wedge q) \vee F(Fp \vee q))))$
(v) $(Gp \rightarrow Fp)$
(vi) $(Fp \rightarrow FFp)$
(vii) $((Fp \wedge FG{\sim}p) \rightarrow F(HFp \wedge G{\sim}p))$

Rules.
Tautological consequence.
Temporal generalization: From φ to infer $G\varphi$ and $H\varphi$.
Mirror image rule: From a formula, you may derive the correspoinding formula obtained by exchangind "G" and "H" and also "F" and "P."

1. Show, by giving a derivation, that "PGp" implies "GPp."
2. Show, by giving a derivation, that "PGp" implies "HFp."
3. Show, by giving a derivation, that "PGp" implies "GFp."
4. Show that "GFp" doesn't imply "PGp" by giving a set S of real numbers such that (assuming lesser numbers correspond to earlier times and 0 corresponds to the present), if "p" is true at the times in S and at no other times, "GFp" will now be true and "PGp" now false.
5. Show, as in problem 4, that "PGp" doesn't imply "HPp."
6. Show, as in problem 4, that "FGp" doesn't imply "GPp."
7. Show that each of the four formulas you obtain by prefixing one of the four operators to "PGp" is equivalent to a formula in the diagram. You

don’t need to do derivations. Just show that, assuming times are ordered like the real numbers, at whatever times “p” is true, any formula obtained by prefixing one of the four operators to “PGp” will be true at the same times as one of the formulas in the diagram.

8. Show that each of the four formulas you obtains by prefixing one of the four operators to “HPp” is equivalent to a formula on the diagram. You don’t need to do derivations. Talk about times instead.
9. Show that each of the for formulas you obtains by prefixing one of the four operators to “FPp” is equivalent to a formula on the diagram. You don’t need to do derivations.

Axioms. We use “Pp” to abbreviate “~H~p” and “Fp” to abbreviate “~G~p.”

(i) $(G(p \supset q) \supset (Gp \supset Gq))$

(ii) $(p \supset GPp)$

(iii) $(Gp \supset GGp)$

(iv) $((Fp \wedge Fq) \supset (F(p \wedge Fp) \vee (F(p \wedge q) \vee F(Fp \vee q))))$

(v) $(Gp \supset Fp)$

(vi) $(Fp \supset FFp)$

(vii) $((Fp \wedge FG{\sim}p) \supset F(HFp \wedge G{\sim}p))$

Rules.

Tautological consequence.

Temporal generalization: From φ to infer Gφ and Hφ.

Mirror image rule: From a formula, you may derive the correspoinding formula obtained by exchangind “G” and “H” and also “F” and “P.”