

Stalnaker Logic of Conditionals

Define a *candidate Stalnaker model* to be a quadruple $\langle W, I, f, @ \rangle$, where

W is a nonempty set of things we're calling worlds;
 I is a function $\{\text{atomic formulas}\} \times W \rightarrow \{T, F\}$;
 f is a partial function $W \times \{\text{formulas}\} \rightarrow W$ (that is, f is a function from a subset of $W \times \{\text{formulas}\}$ to W); and
 $@$, the actual world of the model, is an element of W .

We define what it is for a formula to be true in a world in the model:

An atomic formula α is true in w iff $I(\alpha, w) = T$
A conjunction is true in w iff both conjuncts are true in w .
A disjunction is true in w iff one or both disjuncts are true in w .
A negation is true in w iff the negatum isn't true in w .
" $\perp$ " isn't true in w .
A conditional $(\varphi > \psi)$ is true in w iff either $\langle w, \varphi \rangle$ isn't in $\text{Dom}(f)$ or $\langle w, \varphi \rangle$ is in $\text{Dom}(f)$ and ψ is true in $s(w, \varphi)$.

A formula is true in the model iff it's true in $@$.

A *Stalnaker model* is a candidate Stalnaker model that meets these further conditions:

φ is true in $f(w, \varphi)$.
If φ is true in w , $f(w, \varphi) = w$.
If ψ is true in $f(w, \varphi)$, $\langle w, \psi \rangle$ is in $\text{Dom}(f)$.
If ψ is true in $f(w, \varphi)$ and φ is true in $f(w, \psi)$, $f(w, \varphi) = f(w, \psi)$.

A formula is a *Stalnaker theorem* iff it's derived from the following axioms by the following rules:

Axioms:

(Conditional K) $((\varphi > (\psi \rightarrow \theta)) \rightarrow ((\varphi > \psi) \rightarrow (\varphi > \theta)))$
(Reflexive law) $(\varphi > \varphi)$
(Modus ponens) $((\varphi > \psi) \wedge \varphi) \rightarrow \psi$
(Centering) $((\varphi \wedge \psi) \rightarrow (\varphi > \psi))$
(Equivalent antecedents) $((\varphi > \psi) \wedge (\psi > \varphi)) \rightarrow ((\varphi > \theta) \leftrightarrow (\psi > \theta))$
(Conditional excluded middle) $((\varphi \Rightarrow \psi) \vee (\varphi \Rightarrow \sim \psi))$

Rules:

(Tautological consequence) You may write any tautology or tautological consequence of things you've written earlier.
(Conditionalization). If you've written ψ , you may write $(\varphi \Rightarrow \psi)$.

Soundness Theorem. Every Stalnaker theorem is true in every Stalnaker model.

Completeness Theorem. Every formula that is true in every Stalnaker model is a Stalnaker theorem.

Compactness Theorem. If every finite subset of Γ has a Stalnaker model, there is a Stalnaker model in which all the members of Γ are true.