Lewis Semantics for Conditionals

Stalnaker's semantics supposes that, for each world w and sentence φ, if φ is possible in w, then there is a unique world closest to w in which φ is true. David Lewis[1] objected. First, why can't there be ties? Second, couldn't there be a sequence of φ-worlds getting closer and closer to w, without there being a closest world to w in which φ is true?

The most discussed example is this: Bizet and Verdi weren't compatriots, since Bizet was French and Verdi Italian. If they had been compatriots, they would have both been Frence or both Italian, but there isn't any apparent reason to prefer one possibility over the other. Nonetheless, according to Stalnaker's semantics, one or the other of, "If Bizet and Verdi had been compatriots, they would have been French" and, if Bizet and Verdi had been compatriots, they would have been Italian" is true. Stalnaker doesn't find this outcome especially undesirable, but Lewis does, and he gives an easy modification of the Stalnaker semantics that avoids it. Now a conditional will be true in a world w iff the consequent is true in all the closest world to w in which the antecedent is true.

A *Lewis selection function candidate model* is a triple $\langle W, I, g, @\rangle$, where g is a function taking members of $W \times \{\text{sentences}\}$ to sets of worlds. $(\varphi > \psi)$ is true in w iff ψ is true in every member of $g(w,\varphi)$ in which φ is true. A *Lewis selection function model* is a candidate model that meets these further conditions:

> φ is true in every world in $g(w,\varphi)$.
>
> If φ is true in w, then $g(w,\varphi) = \{w\}$.
>
> If ψ is true in every world in $g(w,\varphi)$ and φ is true in every world in $g(w,\varphi)$, then $g(w,\varphi) = g(w,\psi)$.
>
> Unless $g(w,(\varphi \vee \psi))$ consists either entirely of φ worlds or entirely of ψ world, $g(w,(\varphi \vee \psi)) = g(w,\varphi) \cup g(w,\psi)$.

Given a Stalnaker model $\langle W, I, f, @\rangle$ we get a Lewis selection function model by taking $g(w,\varphi)$ to be $\{f(w,\varphi)\}$ if $\langle w,\varphi\rangle$ is in the domain of f, and letting $g(w,\varphi) = \varnothing$ otherwise.

To adapt Stalnaker's axioms to get a sound and complete system of axioms for the Lewis semantics, replace Conditional Excluded Middle with the following axiom schemata:

(Introduction of disjunctive antecedents) $(((\varphi > \theta) \wedge (\psi > \theta)) \rightarrow ((\varphi \vee \psi) > \theta)))$.

(Elimination of disjunctive antecedents) $(((\varphi \vee \psi) > \sim \varphi) \vee (((\varphi \vee \psi) > \theta) \supset (\varphi > \theta)))$.

Let's look at why elimination of disjunctive antecedents is valid in the Lewis semantics.

[1]*Counterfactuals*. Harvard, 1973.

Suppose that $((\varphi \vee \psi) > \theta)$ is true in w. Then θ is true in every world in $g(w,(\varphi \vee \psi))$. Note that $(\varphi \vee \psi)$ is tautologically equivalent to $(\varphi \vee (\sim \varphi \wedge \psi))$. Hence $g(w, (\varphi \vee \psi)) = g(w, (\varphi \vee (\sim \varphi \wedge \psi)))$. So θ is true in every world in $g(w,(\varphi \vee (\sim \varphi \vee \psi)))$. Because of the fourth condition, there are three cases:

Case 1. $g(w,(\varphi \vee (\sim \varphi \wedge \psi))$ consists entirely of φ worlds. Every world in $g(\varphi)$ is a φ world and hence a $(\varphi \vee (\sim \varphi \wedge \psi))$ world. If follows by the third condition that $g(w,(\varphi \vee (\sim \varphi \wedge \psi))) = g(w,\varphi)$ So θ is true in every world in $g(w,\varphi)$, so $(\varphi > \theta)$ is true in w.

Case 2. $g(w,(\varphi \vee \psi))$ consists entirely of $(\sim \varphi \wedge \psi)$ worlds. So it consists entirely of $\sim \varphi$ worlds. So $((\varphi \vee \psi) > \sim \varphi)$ is true in w.

Case 3. $g(\varphi \vee (\sim \varphi \wedge \psi)) = g(\varphi) \cup g(\sim \varphi \wedge \psi)$. So every world in $g(\varphi)$ is a θ world and $(\varphi > \theta)$ is true in w.

Both introduction and elimination of disjunctive antecedents are derivable in Stalnaker's system.

Letting the values of a selection function be sets of worlds rather than single worlds overcomes Lewis's first objection, but it doesn't address the second one. Here is an example of the limit assumption: I certainly could have been over six feet tall. But there isn't a closest world to the actual world in which I am over six feet tall. Suppose instead that w is a world in which I'm over six feet tall. Say my height in w in w is $(72. + \varepsilon)$ inches. But there is a world, otherwise like the actual world in which my height if $(72 + {}^{\varepsilon}/_{2})$ inches, and that, it appears, is a closer world than w in which I'm over six feet tall. (Neither philosopher's story seems to fit the facts of English usage all that well, since we say things like, "If I were over six feet tall, I could reach the top shelf.")

Lewis's remedy is to replace the selection function with a more sophisticated apparatus for comparing worlds in terms of similarity. With each world w, he associates a set $(w) of "spheres" around w, so that if S in in $(w), any world in S is closer to w than any world outside S. Then he declares $(\varphi > \psi)$ true in w iff one of two things happens:

> There is no sphere in $(w) in which φ is true; or
>
> There is a sphere S in $(w) with the properties that:
>
> > There is a φ-world in S.
> > Any φ world in S is a ψ-world.

The system $(w) of sphere around w is assumed to satisfy the following four conditions:

> $(w) is centered on w: $\{w\} \in$ $(w).
>
> $(w) is nested: If S and T are in $(w), either $S \subseteq T$ or $T \subseteq S$.

Any union of members of $(w) is a member of $(w).

Any intersection of members of $(w) is a member of $(w).

A formula is true in every Lewis sphere model if and only if it's true in every Lewis selection function model.