

24.244. Modal Logic. P-set due Thursday, Nov. 12.

1. Show, by giving a derivation, that  $((\varphi \vee \psi) > \sim \varphi) \vee (\varphi \vee \psi) > \sim \psi \vee (((\varphi \vee \psi) > \theta \leftrightarrow ((\varphi > \theta) \wedge (\psi > \theta)))$  is a theorem of Lewis's system
2. Show that the instances of the schema  $((\varphi > \theta) \wedge (\psi > \theta)) \rightarrow ((\varphi \vee \psi) > \theta)$  are true in every Lewis sphere model.
3. Is " $((P \wedge Q) > R) \rightarrow ((P > R) \vee (Q > R))$ " a theorem of Stalnaker's system? Explain.
4. If " $((P > R) \vee (Q > R)) \rightarrow ((P \wedge Q) > R)$ " a theorem of Stalnaker's system? Explain.
5. The law of exportation is the schema  $((\varphi \wedge \psi) > \theta) \rightarrow (\varphi > (\psi > \theta))$ . Show that, if we add it to Stalnaker's axioms,  $((\varphi > \psi) \leftrightarrow (\varphi \rightarrow \psi))$  will become derivable.
6. The law of importation is the schema  $((\varphi > (\psi > \theta)) \rightarrow ((\varphi \wedge \psi) > \theta))$ . Show that, if we add it to Stalnaker's axioms,  $((\varphi > \psi) \leftrightarrow (\varphi \rightarrow \psi))$  will not become derivable.
7. Let  $\Gamma$  be smallest collection of sentences of the language obtained from the language of arithmetic by adding the new predicate "Nec" that:
  - contains all the consequences of Q and all sentences of the form  $\text{Nec}([\ulcorner \theta \urcorner])$  for  $\theta$  a consequence of Q;
  - contains all sentences of the form  $(\text{Nec}([\ulcorner (\varphi \rightarrow \psi) \urcorner]) \rightarrow (\text{Nec}([\ulcorner \varphi \urcorner] \rightarrow \text{Nec}([\ulcorner \psi \urcorner])))$ ; and
  - contains all sentences of the forms  $(\text{Nec}([\ulcorner \varphi \urcorner]) \rightarrow \varphi)$  and  $\text{Nec}([\ulcorner (\text{Nec}([\ulcorner \varphi \urcorner]) \rightarrow \varphi) \urcorner])$ .
 Show that the formalization of Goldbach's conjecture ("Every even number  $> 2$  is the sum of two primes") is a consequence of  $\Gamma$ .

