Subject 24.244. Modal Logic. Answers to the seventh p-set.
In these problem, $(\varphi \rightarrow \psi)$ is the material conditional. It's true if either φ is false or ψ is true.
$(\varphi > \psi)$ is the Stalnaker conditional. It's true if ψ is true in the closest world (if there is one) where φ is true.

1. **Show that the transitivity schema, $(((\varphi > \psi) \wedge (\psi > \theta)) \rightarrow (\varphi > \theta))$ doesn't follow from Stalnaker's axioms**.
Take a Stalnaker model with two world, @ and w, each of which has access to itself and the other. Let "P" be true in w only, and let "R" be true in @ only, and let "Q" be true in both worlds. The f(@, "P"), which in w, is a "Q"-world that isn't an "R"-world f(@, "Q"), which is @ itself, in an "R"-world. So "$(((P > Q) \wedge (Q > R)) \rightarrow (P > R))$" is false in @.

2. **Show that this restricted version of the transitivity schema does follow from Stalnaker's axioms: $(((\varphi > \psi) \wedge (\psi > \_|\_)) \rightarrow (\varphi > \_|\_))$.**

1. $(\bot \rightarrow \varphi)$	TC
2. $(\psi > (\bot \rightarrow \varphi))$	Conditionalization 1
3. $((\psi > (\bot \rightarrow \varphi)) \rightarrow ((\psi > \bot) \rightarrow (\psi > \varphi)))$	Conditional K
4. $(((\varphi > \psi) \wedge (\psi > \varphi)) \rightarrow ((\varphi > \bot) \leftrightarrow (\psi > \bot)))$	Equivalent antecedents
5. $(((\varphi > \psi) \wedge (\psi > \bot)) \rightarrow (\varphi > \bot))$	TC 2, 3, 4

3. **The law of Duns Scotus is the schema $(\sim \varphi > (\varphi > \psi))$. Are the instances of the schema derivable in Stalnaker's system? Explain your answer.**
No. Take a model with two worlds, @ and w, where @ has access to both worlds and w has access only to itself, and where "P" is true only at w, and "Q"isn't true at either world. Because "Q" is false at w, "(P > Q)" is false at @. Because "P" is false in @, s(@, ~ p) = @, so "(~ p > (q > p))" is false in @, so it's not derivable from Stalnaker's axioms.

4. **Peirce's law is the schema $(((\varphi > \psi) > \varphi) > \varphi)$. Are the instances of the schema derivable in Stalnaker's system? Explain your answer.**
No. Take a model with two worlds, @ and w, each of which as access to both worlds. Suppose "P" is true in w only, and "Q" isn't true in either world. The "(P > Q)" isn't true in either world, so "((P > Q) > P)" is true at @, even thought "P" is false in @. "(((P > Q) > P) > P" is false in @, so not derivable.

5. **The law of exportation is the schema $(((\varphi \wedge \psi) > \theta) \rightarrow (\varphi > (\psi > \theta)))$. Are the instances of the schema derivable in Stalnaker's system? Explain your answer.**
No. Take a model with three worlds, @, w, and v. "P" is true in @ and v. "Q" is true in w and v. "R" is true in @ and v. The nearest world to @ in which "Q" is true in w, and the nearest world to @ in which "P" and "Q" are both true is v, which is farther from @ than w. "$((P \wedge Q) > R)$" is true in @ iff "R" is true win f(@, "$(P \wedge Q)$"), which is v. "R" is true in v, so "$((P \wedge Q) > R)$" is true in @.

"(P > (Q > R))" is true in @ iff "(Q > R)" is true in f(@, "P"), which is @. "(Q > R)" is true in @ iff "R" is true in f(@, "Q"), which is w. "R" isn't true in w. So "$(((P \wedge Q) > R) \rightarrow (P > (Q > R)))$" is false in @, hence not derivable.

"(Q > R)" is true in @ iff "R" is true is f(@, "Q"), which is w. "R" is true in w. So "(P > (Q > R))" is true in @.

6. **The law of importation is the schema $((\varphi > (\psi > \theta)) \rightarrow ((\varphi \wedge \psi) > \theta))$. Are the instances of the schema derivable in Stalnaker's system? Explain your answer.**

No. Take a model with three worlds, @, w, and v. "P" is true in @ and v. "Q" is true in w and v. "R" is true in w only. The nearest world to @ in which "Q" is true in w, and the nearest world to @ in which "P" and "Q" are both true is v, which is farther from @ than w. "((P > (Q > R))" is true in @ iff "(Q > R)" is true win f(@, "P"), which is @. "(Q > R)" is true in @ iff "R" is true is f(@, "Q"), which is w. "R" is true in w. So "(P > (Q > R))" is true in @.

"((P ∧ Q) > R)" is true in @ iff "R" is true in f(@, "(P ∧ Q)"), with is v. "R" isn't true in v. Consequently, "((P ∧ Q) > R)" is false in @. It follows that "((P > (Q > R)) → ((P ∧ Q) > R))" is false in @, so not derivable.

7. **Show, by giving a derivaion, that the Strong Centering schema, $(((\varphi > \psi) \wedge \varphi) \leftrightarrow (\varphi \wedge \psi))$ is a theorem schema of Stalnaker's system**.

1. $(((\varphi > \psi) \wedge \varphi) \rightarrow \psi)$	Modus ponens
2. $((\varphi \wedge \psi) \rightarrow (\varphi > \psi))$	Centering
3. $(((\varphi > \psi) \wedge \varphi) \leftrightarrow (\varphi \wedge \psi))$	TC 1, 2

8. **Show, by giving a derivation, that the schema $(((\varphi \vee \psi) > \sim \varphi) \vee (((\varphi \vee \psi) > \theta) \supset (\varphi > \theta))$ is a theorem schema of Stalnaker's system.**

1. $((((\varphi \vee \psi) > \varphi) \wedge (\varphi > (\varphi \vee \psi))) \rightarrow (((\varphi \vee \psi) > \theta \leftrightarrow (\varphi > \theta)))$	Equivalent antecedents
2. $(\varphi \rightarrow (\varphi \vee \psi))$	TC
3. $(\varphi > (\varphi \rightarrow (\varphi \vee \psi)))$	Conditionalization 2
4. $((\varphi > (\varphi \rightarrow (\varphi \vee \psi))) \rightarrow ((\varphi > \varphi) \rightarrow (\varphi > (\varphi \vee \psi))))$	Conditional K
5. $(\varphi > \varphi)$	Reflexive law
6. $(((\varphi \vee \psi) > \varphi) \vee ((\varphi \vee \psi) > \sim \varphi)$	Conditional excluded middle
7. $(((\varphi \vee \psi) > \sim \varphi) \vee (((\varphi \vee \psi) > \theta) \rightarrow (\varphi > \theta)))$	TC 1, 3, 4, 5, 6

9. **Show, by giving a derivation, that the Introduction of Disjunctive Antecedents schema, $(((\varphi > \theta) \wedge (\psi > \theta)) \rightarrow ((\varphi \vee \psi) > \theta)))$, is a theorem schema of Stalnaker's system.**

1. $(((\varphi > (\varphi \vee \psi)) \wedge ((\varphi \vee \psi) > \varphi)) \rightarrow ((\varphi > \theta) \leftrightarrow ((\varphi \vee \psi) > \theta)))$	Equivalent antecedent
2. $(\varphi \rightarrow (\varphi \vee \psi))$	TC
3. $(\varphi > (\varphi \rightarrow (\varphi \vee \psi)))$	Conditionalization 2
4. $((\varphi > (\varphi \rightarrow (\varphi \vee \psi)) \rightarrow ((\varphi > \varphi) \rightarrow (\varphi > (\varphi \vee \psi))))$	Conditional K
5. $(\varphi > \varphi)$	Reflexive law
6. $(((\varphi \vee \psi) > \varphi) \rightarrow (((\varphi > \theta) \wedge (\psi > \theta)) \rightarrow ((\varphi \vee \psi) > \theta)))$	TC 1, 3, 4, 5
7. $(((\psi > (\varphi \vee \psi)) \wedge ((\varphi \vee \psi) > \psi))) \rightarrow ((\psi > \theta) \leftrightarrow ((\varphi \vee \psi) > \theta)))$	Equivalent antecedents
8. $(\psi \rightarrow (\varphi \vee \psi))$	TC
9. $(\psi > (\psi \rightarrow (\varphi \vee \psi)))$	Conditionalization 8
10. $((\psi > (\psi \rightarrow (\varphi \vee \psi))) \rightarrow ((\psi > \psi) \rightarrow (\psi > (\varphi \vee \psi))))$	Conditional K
11. $(\psi > \psi)$	Reflexive law

12. $(((\varphi \vee \psi) > \psi) \to (((\varphi > \theta \wedge \psi > \theta) \to ((\varphi \vee \psi) > \theta)))$ TC 7, 9, 10, 11
13. $(((\varphi \vee \psi) > \varphi) \vee ((\varphi \vee \psi) > \psi)) \to (((\varphi > \theta \wedge \psi > \theta) \to ((\varphi \vee \psi) > \theta)))$ TC 6, 12
14. $((\varphi \vee \psi) \to (\sim \varphi \to \psi))$ TC
15. $((\varphi \vee \psi) > ((\varphi \vee \psi) \to (\sim \varphi \to \psi)))$ Conditionalization 14
16. $(((\varphi \vee \psi) > ((\varphi \vee \psi) \to (\sim \varphi \to \psi))) \to ((\varphi \vee \psi) > (\varphi \vee \psi)) \to ((\varphi \vee \psi) > (\sim \varphi \to \psi))))$ Conditional K
17. $((\varphi \vee \psi) > (\varphi \vee \psi))$ Reflexive law
18. $((\varphi \vee \psi) > (\sim \varphi \to \psi))$ TC 15, 16, 17
19. $(((\varphi \vee \psi) > (\sim \varphi \to \psi)) \to ((\varphi \vee \psi) > \sim \varphi) \to ((\varphi \vee \psi) > \psi)$ Conditional K
20. $(((\varphi \vee \psi) > \sim \varphi) \to ((\varphi \vee \psi) > \psi))$ TC 18, 19
21. $(((\varphi \vee \psi) > \varphi) \vee ((\varphi \vee \psi) > \sim \varphi)$ Conditional excluded middle
22. $(((\varphi \vee \psi) > \varphi) \vee ((\varphi \vee \psi) > \psi))$ TC 20, 21
23. $(((\varphi > \theta) \wedge (\psi > \theta)) \to ((\varphi \vee \psi) > \theta))$13, 22 TC