Subject 24.244. Modal Logic. Problem set due Thursday, December 3

1. Use Russell's theory of descriptions to distinguish four ways of symbolizing "The last star seen in the morning is necessarily identical to the first star seen in the evening." Which of the four are true? Explain your answer.

2. Give two ways to symbolize "Whatever is square is necessarily rectangular." Are either or both of them true? Explain.

3. We can symbolize "The person who wrote *Harry Potter and the Philosopher's Stone* might not have written *Harry Potter and the Philosopher's Stone*" as "$\Diamond \sim W\iota x(Wx)$." Russell's method gives us three ways to eliminae "$\iota x(Wx)$" from this symbolization. Which, if any, of the three are true? Explain your answer.

4. Use Russell's theory of definite descriptions to give two readings of "The largest prime number is necessairily odd." Which, if either, of them is true? Explain.

5. Do the same for "The smallest prime number is necessarily odd."

6. Show that if, within the system of axioms for predicate logic, the axiom schema (US) is replaced by (EG), we'll get an equivalent system:
(EG) $(\varphi(c) \rightarrow (\exists v_i)\varphi(v_i))$.

7. Show that if, withing the system of axioms for predicate logic, the rule UG is replaced by ES, we'll get an equivalent system:
ES If c doesn't occur within $\varphi(v_i)$ or ψ, then from $(\varphi(c) \rightarrow \psi)$ you may infer $((\exists v_i)\varphi(v_i) \rightarrow \psi)$.

8. Which of the following are theorems of modal predicate logic with constant domains? Explain:
   a) $(\exists x)\Box Fx \rightarrow \Box(\exists x)Fx$.
   b) $\Box(\exists x)Fx \rightarrow (\exists x)\Box Fx$.
   c) $(\forall x)\Diamond Fx \rightarrow \Diamond(\forall x)Fx$.
   d) $\Diamond(\forall x)Fx \rightarrow (\forall x)\Diamond Fx$.
   e) $\Diamond(\exists x)Fx \rightarrow (\exists x)\Diamond Fx$.
   f) $(\exists x)\Diamond Fx \rightarrow \Diamond(\exists x)Fx$.

9. Show that $(\Box\neq)$ isn't derivable from the other axioms.