

The Barcan formula is an axiom. If we don't like it, we can remove it from our list of axioms. Its converse is harder to get rid of. The way we get an axiom system for modal predicate calculus is to take our system of axioms and rules for modal sentential calculus and or system of axioms and rules for the plain predicate calculus and combine them. For the version of the plain predicate calculus without individual constants, this is a typical set of axioms:

(Taut) Every tautological formula.

(US) $(\forall v_i)\varphi(v_i) \rightarrow \varphi(v_j)$, where $\varphi(v_j)$ is like $\varphi(v_i)$, except for containing free v_j at some places where $\varphi(v_i)$ contains free v_i .

($\forall$ Dist) $(\forall v_i)(\varphi \rightarrow \psi) \rightarrow ((\forall v_i)\varphi \rightarrow (\forall v_i)\psi)$.

(Vac) $\varphi \leftrightarrow (\forall v_i)\varphi$, provided v_i isn't free in φ .

($\exists$ Def) $(\exists v_i)\varphi \leftrightarrow \sim (\forall v_i) \sim \varphi$.

(Ref=) $v_i = v_j$.

(Sub=) $v_i = v_j \rightarrow (\varphi(v_i) \leftrightarrow \varphi(v_j))$, where $\varphi(v_j)$ is like $\varphi(v_i)$ except for containing free v_j at some places where $\varphi(v_i)$ has free v_i .

The rules will include:

UG From φ , you may infer $(\forall v_i)\varphi$,

as well as modus ponens. The modal system will include (K) at minimum, and may optionally contain such other axioms as (T) and (4). It will contain Nec as a rule, and hence K and TC as derived rules

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This combination lets us derive the converse Barcan formula (using "x" instead of " x_i " to avoid subscripts):

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| 1. | $(\forall x)\varphi(x) \rightarrow \varphi(x)$ | (US) |
| 2. | $\Box((\forall x)\varphi(x) \rightarrow \varphi(x))$ | Nec 1 |
| 3. | $\Box(\forall x)\varphi(x) \rightarrow \Box\varphi(x)$ | K 2 |
| 4. | $(\forall x)(\Box(\forall x)\varphi(x) \rightarrow \Box\varphi(x))$ | UG 3 |
| 5. | $(\forall x)\Box(\forall x)\varphi(x) \rightarrow (\forall x)\Box\varphi(x)$ | From 4 by ($\forall$ Dist) |
| 6. | $\Box(\forall x)\varphi(x) \leftrightarrow (\forall x)\Box(\forall x)\varphi(x)$ | (Vac) |
| 7. | $\Box(\forall x)\varphi(x) \rightarrow (\forall x)\Box\varphi(x)$ | TC 5, 6 |

This result looks inevitable, but it isn't really. There are many different axiomatizations of the predicate calculus, which get to the same theorems by different paths. Thus if we replace (US) by the following, we get an equivalent axiomatization of the predicate calculus:

($\forall$ US) $(\forall v_j)((\forall v_i)\varphi(v_i) \rightarrow \varphi(v_j))$