

24.244. Modal Logic. P-set due Tuesday, December 5.

1. Show that all instances of the schema $(\forall v_j)((\forall v_i)\varphi(v_i) \rightarrow \varphi(v_j))$ are derivable in free logic.
2. Show that any sentence that doesn't contain constants that's derivable in ordinary predicate calculus is derivable in free logic. Constants will appear within the derivation, but not within the final result.
3. Show that each of the following formulas is valid in KT with variable domains, either by doing a derivation or by showing that the formula is true in each world in each reflexive model:
 - a) $((\forall v_1)\Box P v_1 \rightarrow (\forall v_2)P v_2)$
 - b) $(\forall v_2)(\Box(\exists v_1)R v_1 v_2 \rightarrow (\exists v_1)\Diamond R v_1 v_2)$
4. Let's say a model $\langle W, R, U, D, N, I, @ \rangle$ is *monotone* iff whenever wRv , $D(w) \subseteq D(v)$. Take a language that contains, among other things, the constant c .
 - a) Show that if a model is monotone, every instance of the converse Barcan formula is satisfied by every variable assignment in every world.
 - b) Show that, if $\langle W, R, U, D \rangle$ isn't monotone, then there is a function N assigning a value to the constant c in such a way that $\Box(\exists v_1)c = v_1$ is false at some world in the model $\langle W, R, U, D, N, I, @ \rangle$.
 - c) Show that, if $\langle W, R, U, D \rangle$ isn't monotone, then there is an instance of the converse Barcan formula that's false at some world in a model $\langle W, R, U, D, N, I, @ \rangle$.
5. Let's say a model $\langle W, R, U, D, N, I, @ \rangle$ is *anti-monotone* iff whenever wRv , $D(v) \subseteq D(w)$. Take a language that contains, among other things, the constant c .
 - a) Show that if a model is anti-monotone, every instance of the Barcan formula is satisfied by every variable assignment in every world.
 - b) Show that, if $\langle W, R, U, D \rangle$ isn't anti-monotone, then there is a function N assigning a value to the constant c in such a way that $(\Diamond(\exists v_1)c = v_1) \rightarrow (\exists v_1)c = v_1$ is false at some world in the model $\langle W, R, U, D, N, I, @ \rangle$.
 - c) Show that, if $\langle W, R, U, D \rangle$ isn't anti-monotone, then there is an instance of the Barcan formula that's false at some world in a model $\langle W, R, U, D, N, I, @ \rangle$.

