Presuppositions Projection – the Challenge

1. Simple Theory of Projection – Cumulative[1]

(1) **Cumulative Hypothesis (CH)**: If ϕ is a (matrix) sentence that dominates a constituent S with presupposition p, then p is a presupposition of ϕ. (Gazdar 1979)

(2) a. The king of France isn't bald.
b. Is the king of France bald?

(3) **CH in Trivalent Semantics**: If ϕ is a (matrix) sentence that dominates a constituent X such that $[\![X]\!]= \#$, then $[\![\phi]\!]= \#$ (otherwise compute denotation based on bivalent recipe).

Challenges

- Cases where CH appears to make wrong predictions.
- Cases where predictions should be made, but CH makes no prediction at all.

1.1. Filters

The sentences below have no presupposition although they contain a constituent that carries a presupposition. They thus appear to refute the cumulative hypothesis.

(4) a. If this house has a kitchen, the kitchen is on the second floor.
b. Either this house has no kitchen, or the kitchen is on the second floor.
c. (I don't think that) John was here before and will be here again in the future.

(5) **Karttunen's Empirical Claim**:
Let S_p be a sentence such that p is its semantic presupposition and let A be a sentence with no presupposition:
a. Presupposition(*If A then* S_p) = $\{w: [\![A]\!]^w = 1 \rightarrow p(w) =1\}$
b. Presupposition(*Either A or* S_p) = $\{w: [\![A]\!]^w = 0 \rightarrow p(w) =1\}$
c. Presupposition(*A and* S_p) = $\{w: [\![A]\!]^w = 1 \rightarrow p(w) =1\}$

Gazdar's Reply: The cumulative hypothesis is correct, but there is a mechanism of "presupposition cancellation", *which can apply if a presupposition leads to pragmatic anomaly.*

(6) a. There is no king in France. So, of course, the king of France isn't bald.
b. The King of France doesn't exist.

[1] Most of the discussion in this section follows that in Heim (1990), http://semanticsarchive.net/Archive/GFiMGNjN/Presupp%20projection%2090.pdf

In (6) the predicted presupposition (if it was not cancelled) would contradict the assertion. (In a trivalent system we would have a sentence, or sequence of sentences, that is always false or undefined.) In (4), we would have a condition on language use that cannot be met. (More on the relevant conditions of use when we discuss SIs. But, for now, we might say that certain sentences are associated with "ignorance inferences" that might be incompatible with the presupposition. For example, it is a condition on the utterance of a conditional that the speaker is ignorant about the truth value of the antencedent. This condition would have to be violated if the presupposition of the consequent in (4)a were common ground.)

(7) **Soams' Observation**: there are cases where no anomaly is predicted under the cumulative hypothesis, yet the predictions are wrong.
   a. If you watch this movie, you will never watch a movie again. (Heim 1990)
   b. If someone in our agency sold a 50 million dollar house today, the daily earnings might allow us to survive a few more weeks. [If no one sold anything, we will close shop tomorrow…]
   c. If John is a scuba diver, he'll bring his wetsuit. (Geurts 1996)
   (Appears to presuppose: If John is a scuba diver, he has a wetsuit. Or maybe, Scuba divers have wetsuit.)
   d. (John thinks that France is ruled by a benevolent king. Moreover) He is sure that the king of France is going to maintain universal healthcare under all possible circumstances.

Conclusion: There seems to be evidence for the existence of "filters" and, hence, against the cumulative hypothesis.

But note that a strategy of presupposition cancellation still needs to be assumed even if Karttunen's statements are correct. Besides (6), it has been claimed to be needed for the following case from Soames:

(8) He looks very nervous. There is only one possible explanation. He either just stopped smoking or just started smoking.

Question:
-What are the projection rules?
-What explains the projections rules?

1.3. A short aside about the Proviso problem

(9) a. If John is a scuba diver, he will bring his wetsuit.
   Inference: If John is a scuba diver, he has a wetsuit.
   b. If John flies to London, his sister will pick him up.
   Inference: John has a sister.

(10)a. Either John is not a scuba diver or he will bring his wetsuit.
   Inference: If John is a scuba diver, he has a wetsuit.

b. Either John is not a scuba diver or his car has a wetsuit in it.
   Inference: John has a car.

(11)a. If x lands on planet W, x will realize that he weighs less than on earth.
   Inference: If x lands on planet W, x will weigh less than on earth.
b. If x weighs himself, x will realize that he weighs less than on earth.
   Inference: x weighs less than on earth.

Standard Response: There is nothing wrong with the semantic presupposition (the minimal condition that the common ground must meet), but there is a problem for an algorithmic view of accommodation (or for our definition of accommodation from last week).

Let ψ_p be a sentence that has p as its presupposition when uttered in isolation and φ be a sentence that has no presupposition.

Weak Presuppositions + Pragmatic Strengthening:
Presupposition(φ *or* ψ_p) = φ *or* $p = \neg\varphi \rightarrow p$
Presupposition(*if* φ *then* ψ_p) = $\neg\varphi$ *or* $p = \varphi \rightarrow p$

The stronger inference, *p*, occurs in communicative situations where it is implausible for the disjunction to be part of the common ground without *p* being part of the common ground.

(12) **Presupposition Accommodation**:
-When a speaker utters S_p, there must be some proposition p' that entails p, such that the speaker intends p' to be taken for granted. If no such proposition is entailed by the common ground, the hearer might nevertheless agree (for the purpose of the conversation, or for real) to play along, which will involve trying to figure out what p' is.
-The speaker might be in a position to anticipate the hearer's cooperative stance and to use S_p when she knows its presupposition is not common ground.

In either case, the context-set will be updated ***by some proposition that entails*** p before it is updated by S_p.

In other words, any accommodation is permissible as long as it is a member of the following set:
Accommodation(C, S) = $\{C \cap p: p \subseteq \text{Presupposition}(S)\}$

More soon (maybe)! You might want to read Winter 2019, 2020 for a recent alternative.

1.4. Presuppositions in the scope of a variable binder

(13)Every/Some/No girl [1 t_1 drove her_1 car to school].
Fact: Presupposes something, either that every girl or, at least, that some girl has a car.
Predictions of Cumulative Hypothesis: no prediction is made.
Question: what is the trivalent denotation of the sister of the subject in (13)?

1. λx: x has a unique car. x drives x's car to school

Total extensions of 1:

2. λx. x has a unique car and x drives x's car to school

2'. λx. x has no car or x has unique car and x drives x's car to school

2. Stipulations for Higher Order Functions

(14)**Heim and Kratzer's limited cumulativity**: Semantics is partial, and constituents might have an undefined semantic value. If a constituent has an undefined semantic value, then this will be inherited by constituents that dominate it.

(14) says nothing about higher order functions. We might capitalize on this to correct for our wrong predictions for the connectives, e.g.:

2.1. Connectives

(15) **Lexical entries for the connectives (designed to capture Karttunen's claim)**:

a. $[\![\text{if then}]\!] = \lambda w.\lambda q.\lambda p\text{: } p(w) \neq \# \ \& \ [p(w)=1 \rightarrow q(w) \neq \#]. \ P(w)=1 \rightarrow q(w) =1.$

b. $[\![\text{or}]\!] = \lambda w.\lambda q.\lambda p\text{: } p(w) \neq \# \ \& \ [p(w)=0 \rightarrow q(w) \neq \#]. \ P(w)=1 \text{ or } q(w) =1.$

c. $[\![\text{and}]\!] = \lambda w.\lambda q.\lambda p\text{: } p(w) \neq \# \ \& \ [p(w)=1 \rightarrow q(w) \neq \#]. \ P(w)=q(w)=1.$

But this is entirely devoid of insight. The moment we introduce the possibility that the argument of a function could be itself a partial function, there are multiple different hypotheses we could entertain. And this is just one of many other possibilities. See discussion in Heim 1990 and Schlenker 2008.

2.2. Generalized Quantifiers

Consider any generalized quantifier, e.g. (16).

(16) $[\![\text{Every girl}]\!]^w = \lambda P_{et}. \ \forall x \ ([\![\text{girl}]\!]^w(x)=1 \rightarrow P(x) =1)$

(16) was written with bivalent semantics in mind. But what happens when the argument of (16) is a partial/trivalent function (e.g. the denotation of VP/T' in (13), *λx: x has unique car. x drives x's car to school*)? (14) is of no help and there are multiple different hypotheses/stipulations we could entertain:

(17) **Possible ways to state the lexical entry of a QP in trivalent semantics:**[2]

a. $[\![\text{Every girl}]\!]^w = \lambda P_{et}: \forall x \in D_e(x \in \text{Domain}(P)).$
$\forall x([\![\text{girl}]\!]^w(x)=1 \rightarrow P(x)=1)$

b. $[\![\text{Every girl}]\!]^w = \lambda P_{et}: \forall x([\![\text{girl}]\!]^w(x)=1 \rightarrow x \in \text{Domain}(P)).$
$\forall x([\![\text{girl}]\!]^w(x)=1 \rightarrow P(x)=1)$

c. $[\![\text{Every girl}]\!]^w = \lambda P_{et}: \exists x([\![\text{girl}]\!]^w(x)=1 \;\&\; x \in \text{Domain}(P)).$
$\forall x([\![\text{girl}]\!]^w(x)=1 \rightarrow P(x)=1)$

d. $[\![\text{Every girl}]\!]^w = \lambda P_{et}. \forall x([\![\text{girl}]\!]^w(x)=1 \rightarrow P(x)=1)$

e. $[\![\text{Every girl}]\!]^w = \lambda P_{et}: \exists x([\![\text{girl}]\!]^w(x)=1 \;\&\; x \in \text{Domain}(P)).$
$\forall x([\![\text{girl}]\!]^w(x)=1 \rightarrow P(x) \neq 0)$

f. $[\![\text{Every girl}]\!]^w = \lambda P_{et}. \forall x([\![\text{girl}]\!]^w(x)=1 \rightarrow P(x) \neq 0)$

…

We might think that the lexicon needs to make specific stipulations for every higher order function, but are we missing a generalization? For example, could there be a single statement that would tell us how our classical bivalent entries for higher order functions should deal with the partiality/trivalence of their arguments?

3. The notion of Potential Relevance and Universal Projection

Question: If the argument of $[\![\text{Every girl}]\!]$ is a partial function, why might we care whether it is defined (receives a bivalent value) for individuals that are in the restrictor of the quantifier and we definitely do not care whether it is defined for other individuals in the domain of discourse? In other words, what is so special about the restrictor of a quantifier?

Possible Answer: Individuals that the restrictor is true of are precisely those individuals that *might be relevant* for evaluating the truth-value of the quantificational statement.

(18) x of type e *might be relevant* for Q of type <et,t>, Rel(x, Q), iff the question of whether the P argument of Q is true of x can have consequences for the truth value of Q(P). That is, Rel(x, Q) iff
$\exists P\, \exists P'$ (P and P' are total/bivalent functions of type <e,t> & $[\forall y \neq x\; P(y)=P'(y)]$ &
$[Q(P) \neq Q(P')]$)

(19) $\text{Rel}(x_e, [\![\text{Every girl}]\!]^w)$ iff $[\![\text{girl}]\!]^w(x)=1$ (Exercise: Prove!)

(20) Conjecture: If there is a unique minimal set in $\{A: \text{Lives-on}(Q,A)\}$, $A_{min,}$ then $\text{Rel}(x_e, Q)$ iff $x \in A_{min}$
(Where Lives-on(Q,A) iff $\forall B[Q(B) \leftrightarrow Q(B \cap A)]$)
(Exercise: prove or refute)

[2] If we adopt a trivalent setup all functions are total, so by Domain(P) we mean the bivalent-domain, i.e. $\{x \in D_c : P(x) \neq \#\}$.

(21)**A way of stating "universal projection"**

Let Q be a bivalent function of type <et,t> (i.e. defined only for bivalent predicates of type et). The trivalent re-writing/extension of Q, Q^t, is the following:

$$Q^t = \lambda P_{et}: \forall x_e \text{ Rel}(x, Q) \rightarrow P(x) \neq \#.\ \exists P'[(P' \text{ is a total extension of } P) \ \&\ Q(P')=1]$$
$$= \lambda P_{et}: \forall x_e \text{ Rel}(x, Q) \rightarrow P(x) \neq \#. \forall P'[(P' \text{ is a total extension of } P) \rightarrow Q(P')=1]$$

Exercise: Prove that equality holds here by showing that if P', P'' are total extensions of P and $\forall x[\text{Rel}(x, Q) \rightarrow (P(x) \neq \#)]$, then Q(P') = Q(P'')

Terminology: We will say that Q of type <et,t> is *uniform across extensions of* P (of type <e,t>) if $\forall P',P''$ [(P' and P'' are both total extensions of P) $\rightarrow$ Q(P')=Q(P'')]

Notational Convention: Whenever Q is uniform across extensions of P, we will simply write Q(P) for the result of applying Q to an arbitrary extension of P. Hence:

(22)**Stating universal projection with use of Notational Convention**

Let Q be a bivalent function of type <et,t>, the trivalent re-writing of Q, Q^t, is the following:

$$Q^t = \lambda P_{et}: \forall x_e[\text{Rel}(x, Q) \rightarrow P(x) \neq \#].Q(P)$$

Remaining Issues:

a. Empirical Problem: Universal projection seems too strong for certain quantifiers

(23) Some girl drives her car to school.

b. Question: What about other filters? Can this consideration of relevance be modified to yield a general projection principle?

Homework #1: Consider the following lexical entry for *some girl*.

(24) $[\![\text{Some girl}]\!]^w = \lambda P_{et}: \exists x([\![\text{girl}]\!]^w(x)=1 \ \&\ P(x)=1) \text{ iff } \exists x([\![\text{girl}]\!]^w(x)=1 \ \&\ P(x) \neq 0).$
$\exists x([\![\text{girl}]\!]^w(x)=1 \ \&\ P(x)=1)$

a. Explain the predicted presupposition for the sentence in (23).
b. Try to generalize (24) along the lines of (21) by providing values for A, B and C in (25).

(25)**A way of generalizing (24) to all generalized quantifiers**

Let Q be a bivalent function of type <et,t>, the trivalent re-writing/extension of Q, Q^t, is the following:

$$Q^t = \lambda P_{et}: Q(A)=Q(B).\ Q(C)=1$$

c. Explain the predicted presupposition for the sentences *Every/No girl drives her car to school*. How do they differ from the predictions of (21) and which seem more accurate?

Homework #2: Consider the following revisions of (18) and (21):

(26) x of type e *is relevant* given Q of type <et,t> and a (possibly partial/trivalent) function, P, of type <e,t>, Rel(x, Q, P), iff the value that P assign to x has a chance of affecting the truth value of Q(P) [given the values that P assigns to other individuals]. that is Rel(x, Q, P) iff
$\exists P' \exists P''$ (P' and P'' are total extensions/correction of P & $[\forall y \neq x\ P'(y)=P''(y)]$ & $[Q(P) \neq Q(P')]$)

(27) **A possible Projection Recipe**
Let Q be a bivalent function of type <et,t> (i.e. defined only for bivalent predicates of type et). The trivalent re-writing/extension of Q, Q^t, is the following:

$$Q^t = \lambda P_{et}\text{: } \forall x_e\ Rel(x, Q, P) \rightarrow P(x) \neq \#.\ \exists P'[(P' \text{ is a total extension of } P)\ \&\ Q(P')=1]$$
$$= \lambda P_{et}\text{: } \forall x_e\ Rel(x, Q, P) \rightarrow P(x) \neq \#. \forall P'[(P' \text{ is a total extension of } P) \rightarrow Q(P')=1]$$
$$= \lambda P_{et}\text{: } Q \text{ is uniform across extensions of } P.\ Q(P)$$ [understood according to the notational convention introduced above (22)].

a. Explain why the three possible definitions here are identical.
b. Explain what presuppositions are predicted for our three sentences: *Some/Every/No girl drives her car to school*.