## Chierchia's Puzzle and Sauerland's Algorithm

## 1. Deriving Basic Implicatures (very rough outline)

The Neo Gricean Maxim of Quantity (NGMQ) requires a speaker to use the most informative (contextually strongest) sentence from a designated set, $\operatorname{ALT}(\varphi, \mathrm{C})$.

Under various circumstances, (1)' would be a member of the designated set for (1).
(1) John did some of the homework.
(1)' John did all of the homework.

Under certain circumstances, NGMQ allows the listener to infer (upon hearing (1)) that, unless the speaker believed that (1)' were false, the speaker would have uttered (1)'.

Since the speaker didn't make this alternative utterance, it fallows that the speaker believes that (1)' is false.

Derived SI: (S believes) it's not the case that John did all of the homework.

### 1.2. Chierchia's Problem

(2) John did the reading or some of the homework.

Under certain circumstances, NGMQ should allow the listener to infer (upon hearing (2)) that unless the speaker believed that (2)' were false, the speaker would have uttered (2)':
(2)' John did the reading or all of the homework.

Since the speaker didn't make this alternative utterance, it should follow that the speaker believes that (2)' is false.

Derived SI: (S believes) it's not the case that John did the reading or all of the homework.
Chierchia's Problem: $\neg(p \vee q)$ is equivalent to $\neg p \wedge \neg q$; although we get the correct implicature that John didn't do all of the homework, we also get the incorrect implicature that John didn't do the reading.

The problem generalizes to all disjunctions of the form $\mathrm{p} \vee \mathrm{q}$, where q has a stronger alternatives $\mathrm{q}^{+}$.

Chierchia's Generalization: The (pragmatically) strengthened meaning of a disjunctive sentence, $\varphi \vee \psi$, is the following: ${ }^{1}$
$\mathrm{S}(\varphi \vee \psi)=[\mathrm{S}(\varphi) \wedge \neg \psi]$ or $[\mathrm{S}(\psi) \wedge \neg \varphi]$

### 4.3. Sauerland's Proposal ${ }^{2}$

1. Enrich the set of alternatives for disjunction:
(3) Horn-Scale $($ or $)=\{$ or $, L, R$, and $\}(\mathrm{pLq}=\mathrm{p} ; \mathrm{pRq}=\mathrm{q})$
(4) $\operatorname{Alt}(p \vee q)=p \vee q$

2. An algorithm for computing scalar implicatures in response to the utterance of $A:^{3}$
A. Form the set of Primary (weak) Implicatures:

$$
\mathrm{PI}=\left\{\neg \mathrm{B}_{\mathrm{s}}\left(\mathrm{~A}^{\prime}\right): \mathrm{A}^{\prime} \in \mathrm{ALT}(\mathrm{~A}) \text { and } \mathrm{A}^{\prime} \text { is stronger than } \mathrm{A}\right\}
$$

B. Form the set of Secondary (strong) Implicatures:

$$
\begin{aligned}
& \mathrm{SI}=\left\{\mathrm{B}_{\mathrm{s}}\left(\neg \mathrm{~A}^{\prime}\right): \mathrm{A}^{\prime} \in \mathrm{ALT}(\mathrm{~A}), \mathrm{A}^{\prime} \text { is stronger than } \mathrm{A},\right. \text { and } \\
& \left.\mathrm{B}_{\mathrm{s}}(\mathrm{~A}) \wedge \Lambda \mathrm{PI} \wedge \mathrm{~B}_{\mathrm{s}}\left(\neg \mathrm{~A}^{\prime}\right) \text { is not contradictory }\right\}
\end{aligned}
$$

Set A is the set of inferences that we derive directly from (the assumption that the speaker obeys) NGMQ.

Set B is the set of inferences that stronger alternatives are false, which are introduced whenever it is possible to assume that the speaker is opinionated about a stronger alternative; whenever the assumption of an opinionated speaker is consistent with what has already been inferred from the maxim of quantity (but this is a point where you've been encouraged, in the homework, to think more seriously about stuff).

When a simple disjunctive sentence, $p v q$, is uttered, we derive ignorance inferences and SIs as follows:

$$
\begin{aligned}
& \mathrm{PI}=\neg \mathrm{B}_{\mathrm{s}}(\mathrm{p}), \neg \mathrm{B}_{\mathrm{s}}(\mathrm{q}), \neg \mathrm{B}_{\mathrm{s}}(\mathrm{p} \wedge \mathrm{q}) \\
& \mathrm{SI}=\mathrm{B}_{\mathrm{s}} \neg(\mathrm{p} \wedge \mathrm{q})
\end{aligned}
$$

[^0]The disjunction puzzle arises from embedding a non-maximal scalar item under disjunction ( $\mathrm{P} \vee \mathrm{Q}$ where $\mathrm{Q}^{+} \in \operatorname{Alt}(\mathrm{Q})$ and $\mathrm{Q}^{+}$is stronger than Q$)$. The challenge is:

1. To avoid the implicature $\neg\left(\mathrm{P} \vee \mathrm{Q}^{+}\right)$.
2. To derive the implicature $\neg \mathrm{Q}^{+}$

Challenge 1 is met in the following way: $\neg \mathrm{B}_{\mathrm{s}}(\mathrm{Q})$ is a primary implicatures and $\mathrm{B}_{\mathrm{s}}(\mathrm{P} \vee \mathrm{Q}) \wedge \neg \mathrm{B}_{\mathrm{s}}(\mathrm{Q}) \wedge \mathrm{B}_{\mathrm{s}}\left(\neg\left(\mathrm{P} \vee \mathrm{Q}^{+}\right)\right)$is contradictory.

Challenge 2 is met straightforwardly: given (3), $\mathrm{Q}^{+} \in$ Alt $(\mathrm{P} \vee \mathrm{Q})$, and the Secondary Implicature $\mathrm{B}_{\mathrm{s}}\left(\neg \mathrm{Q}^{+}\right)$is consistent with the assertion and the primary implicatures.

In detail
(5) Kai ate some of the soup or the broccoli,
$\operatorname{ALT}(5)=$ ssvb

$\mathrm{B}_{\mathrm{s}}(\mathrm{ss} \vee \mathrm{b})$
$\mathrm{PI}=\neg \mathrm{B}_{\mathrm{s}}(\mathrm{as} \vee b), \neg \mathrm{B}_{\mathrm{s}}(\mathrm{ss}), \quad$ (the rest follow)
$\mathrm{SI}=\mathrm{B}_{\mathrm{s}}(\neg \mathrm{as}), \mathrm{B}_{\mathrm{s}}(\neg(\mathrm{ss} \wedge \mathrm{b})) \quad$ (the rest $\left[\mathrm{B}_{\mathrm{s}}(\neg(\mathrm{as} \wedge \mathrm{b}))\right]$ follows $)$

### 4.4. Further Advantage of the Proposal: embedding under universal quantifiers

(6) You're required to talk to Mary or Sue.

Implicatures:
a. You're not required to talk to Mary.
b. You're not required to talk to Sue.
(7) Every friend of mine has a boy friend or a girl friend.
a. It's not true that every friend of mine has a boy friend.
b. It's not true that every friend of mine has a girl friend.

These facts follow straightforwardly from the Sauerland scale: ${ }^{4}$
(8) $\operatorname{Alt}(\forall x(P(x) \vee Q(x))=\forall x(P(x) \vee Q(x))$
$\mathrm{PI}=\neg \mathrm{B}_{\mathrm{s}}(\forall \mathrm{xP}(\mathrm{x})), \neg \mathrm{B}_{\mathrm{s}}(\forall \mathrm{xQ}(\mathrm{x})) \quad$ (the rest follows)
$\mathrm{SI}=\mathrm{B}_{\mathrm{s}}(\neg \forall \mathrm{xP}(\mathrm{x})), \mathrm{B}_{\mathrm{s}}(\neg \forall \mathrm{xQ}(\mathrm{x})) \quad$ (the rest follows)
Homework \#3: Compute the results of Sauerland's algorithm under the assumption that some is among the lexically specified alternatives of every. What conclusions might you draw from this computation?

### 4.5. The Predicted Generalization

(9) The S-Exhaustivity Generalization (predicted by Sauerland's Theory): utterance of a sentence, A, as a default, licenses the inference that (the speaker believes that) a sentence is false if it is Sauerland-Excludable given A and $\operatorname{Alt}(\mathrm{A})$.
p is Sauerland-Excludable given A and C if $\mathrm{p} \in \mathrm{C}, \mathrm{p}$ is stronger than A and
$\neg \exists \mathrm{q} \in \mathrm{C}[(\mathrm{q}$ is stronger than A$)$ and $(\mathrm{A} \wedge \neg \mathrm{p}$ entails q$)]$.
Homework \#4: Let p be a member of the set ALT(A).
Prove that Sauerland's algorithm yields $\mathrm{B}_{\mathrm{s}}(\neg \mathrm{p})$ as a secondary implicature iff p is Sauerland-Excludable given A and $\operatorname{Alt}(\mathrm{A})$
(I.e., prove that (9) is indeed predicted by Suaerland's theory.)

## 5. Questions:

1. What are L and R ?
2. Is there an intensional definition of alternatives, and in particular does it give us Sauerland's alternatives without postulating L and R.

## 6. Katzir (2007)

### 6.1. Alternatives can be defined by reference to Structural Complexity

(10) $\operatorname{ALT}(\varphi)=\{\psi: \psi$ is as simple (structurally) as $\varphi\}=\{\psi: \psi \leq \varphi\}$
(11) Structural Simplicity: $\leq$ is the minimal relationship such that
a. $\psi \leq \varphi$ if $\psi$ can be derived from $\varphi$ by substitution of one lexical item for another.

[^1]b. $\psi \leq \varphi$ if $\psi$ can be derived from $\varphi$ by substitution of a phrase with a sub-phrase
c. $\psi \leq \varphi$ if $\exists \mu(\psi \leq \mu$ and $\mu \leq \varphi)$.

### 6.2. Matsumoto (1995)

(12) Yesterday it was warm. And today, it's a little bit more than warm.

SI: Yesterday, it wasn't a little bit more than warm.

### 6.3. Katzir's response

(13) Let $A$ be a set of sentences (salient in the surrounding discourse) and let $\varphi \in A$ be an asserted sentence, $\operatorname{ALTA}(\varphi)=\{\psi: \psi \leq A \varphi\}$
(14) Structural Simplicity: $\leq A$ is the minimal relationship such that
a. $\psi \leq A \varphi$ if $\psi$ can be derived from $\varphi$ by substitution of one lexical item for another.
b. $\psi \leq A \varphi$ if $\psi$ can be derived from $\varphi$ by substitution of a phrase with a phrase that is dominated by a member of $A$.
c. $\psi \leq A \varphi$ if $\exists \mu(\psi \leq A \mu$ and $\mu \leq A \varphi)$.
(15) Explain the following facts
a. John did some of the homework. \#And/But Bill did some but not all of the homework.
b. The first detective is sure that the burglars stole some of the jewelry. And the second detective is sure that they stole some but not all of the jewelry.
c. John did some of the homework. Bill also did some but not all of the homework.
6.4. Deriving the theory of alternatives from the maxims (Heim's rendition a few years ago in class of one of the proposals that Katzir entertains)
(16) Normative Principles of language use (revised)
i. Maxim of Quality: s shouldn't utter $\varphi$ if $\neg \mathrm{B}_{\mathrm{s}}(\varphi)$
ii. Maxim of Relevance: $s$ shouldn't utter $\varphi$ if $\varphi$ is not relevant for the topic of conversation.
iii. Maxim of Manner: s shouldn't utter $\varphi$ unless $\varphi$ is equally simple or simpler than a contextual standard (given a set of salient sentences).
iv. Neo-Gricean Maxim of Quantity (NGMQ): s shouldn't utter $\varphi$ if, $\exists \varphi^{\prime}\left[\mathrm{B}_{\mathrm{s}}\left(\varphi^{\prime}\right)\right.$ and $\neg\left(\varphi \Rightarrow_{c} \varphi^{\prime}\right)$ ] and uttering $\varphi$ obeys (i-iv)

## (17) Setting the Contextual standard

Whenever a sentence $\varphi$ is uttered, the contextual standard is adjusted to be the level of complexity of $\varphi$ (given a set of salient sentences).

## 7. Questions from previous handout

i. By revising the maxim of Quantity, have we lost our account for the letter of recommendation?
ii. Have we lost our account of ignorance inferences?

No!
iii. NGMQ is a departure from what seems self evident, namely MQ (in fact it involves a denial of what we called a virtual truism). What could we do if we insisted that MQ is correct? (See Fox 2007, 2014.)
iv. Is there a real generalization here? In particular, can we provide an intensional definition of ALT, which will allow us to say what is predicted independently of lexical stipulations?
Yes but of course understanding the generalization requires knowledge of the syntax of the language
v. Can we create contexts where the relevant maxims are not active and how would that affect the resulting SIs? (Fox 2014)
vi. Are there embedded/intrusive SIs? (Cohen 1971, i.a.)
vii. Are there other areas of language (Grammar or Pragmatics) where similar generalizations to the one we stated here are observed, and what can we learn from the relevant generalizations (Chierchia 2006, Fox and Hackl 2006, Landman 2000, Crnic 2013,...)
viii. We've defined $\operatorname{PI}(\varphi, \mathrm{C})$ with reference to contextual entailment and this followed from the maxim of quantity. Can we check if this is the relevant notion (Fox and Hackl 2006, Hirschberg 1985, Magri 2009, 2011, 2017)? What would we do if we discovered that the relevant notion is one of logical entailment?
ix. We did not refer to probabilities in the statement of the generalization, at least not of PI. If the reasoning that leads to SIs is probabilistic (and takes prior probabilities into account, as in e.g. RSA), does this mean that the generalization is wrong (see Fox and Katzir 2019)? Conversely, what are the consequences for probabilistic models, if the generalization is correct?


[^0]:    ${ }^{1}$ This generalization does not hold for cases in which $\psi$ entails $\varphi$, such as some or all to which we will return.
    ${ }^{2}$ Benjamin Spector $(2005,2007)$ made the same proposal, in a somewhat different format (arguably more general). A related proposal can be found in Yae-Sheik Lee (1995).
    ${ }^{3}$ We only compute implicatures for members of ALT(A) that are relevant in the context of utterance. I ignore this here to reduce clutter.

[^1]:    ${ }^{4}$ As long as we don't generate alternatives for every, an issue we will return to. For relevant discussion, see Fox (2007) and Chemla and Romoli (2015), Crnci et. al. (2015), Bar-Lev and Fox (2017, note 7).

