Algebraic Techniques and Semidefinite Optimization

6.256/18.456 - Spring 2021 - Syllabus

This research-oriented course will focus on algebraic and computational techniques for optimization problems involving polynomial equations and inequalities, and their connections with semidefinite optimization. The goal is to provide a unifying algebraic framework and numerical approaches of general applicability.

The course will develop in a parallel fashion several algebraic and numerical approaches, with a view towards methods that simultaneously incorporate both elements. We will study both the complex and real cases, developing techniques of general applicability, and stressing convexity-based ideas, complexity results, and efficient implementations.

Examples will span several domains, including statistics and systems theory applications. Specific topics include semidefinite programming, resultants and discriminants, hyperbolic polynomials, graphical models, Groebner bases, and sums of squares.

Time and place: Wednesdays and Fridays 1PM-2:30PM, (Zoom, see Canvas site for link)

`canvas.mit.edu/courses/7568`

Instructor: Prof. Pablo A. Parrilo, 32D-724, tel. (617) 324-1542,
e-mail: `parrilo@mit.edu`, URL: `www.mit.edu/~parrilo`.

I am available to meet with students by appointment.

Teaching Assistant: TBA

Prerequisites: Besides general mathematical maturity, the minimal suggested requirements for the course are the following: linear algebra (e.g., 18.06 / 18.700), a background course on linear optimization or convex analysis (e.g., 6.251 or 6.255, 6.253), basic probability (6.041/431), or combinatorial optimization (18.453). Familiarity with the basic elements of modern algebra (e.g., groups, rings, fields), and the essentials of dynamical systems and control (e.g., 6.241) are encouraged, but not required.

Bibliography: We will use a variety of book chapters and current papers. Many of these are listed at the end of this syllabus.

Lecture notes: All handouts, including homework, will be posted in the course website:

`canvas.mit.edu/courses/7568`

Requirements: Research project 50%, Homework 50%.

Homework: Problem sets will be handed out in an approximately biweekly basis, and will be due one week later, at the *beginning of the lecture* on their respective due dates.

We expect you to turn in all completed problem sets on time. Late homework *will not be accepted*, unless there is a prior arrangement with the instructors.

Collaboration policy: We encourage working together whenever possible: in the tutorials, problem sets, and general discussion of the material and assignments.

Keep in mind, however, that for the problem sets the solutions you hand in should reflect your *own* understanding of the class material. It is *not* acceptable to copy a solution that somebody else has written.

Course Syllabus (Preliminary. Dates to be updated.)

Lec.	Time	Topic	Readings
1.		Introduction / Presentation / Review	
2.		Semidefinite programming (I)	
3.		Semidefinite programming (II)	
4.		Algebra review	
5.		Univariate polynomials	
6.		Resultants and discriminants	
7.		Hyperbolic polynomials	
8.		SDP representability	
9.		Newton polytopes/BKK bound	
10.		Sums of squares (I)	
11.		Sums of squares (II)	
12.		SOS Applications	
13.		Varieties, Ideals	
		No classes (Spring break)	
		No classes (Spring break)	
14.		Groebner bases, Nullstellensatz	
15.		Zero dimensional systems (I)	
16.		Zero dimensional systems (II)	
17.		Quantifier elimination	
18.		Infeasibility certificates, Real Nullstellensatz	
19.		Representation theorems	
		No classes (Patriot's day)	
20.		Symmetry reduction methods	
21.		Apps: polynomial solving, Markov chains	
22.		Graph theoretic methods	
23.		Advanced topics	
24.		Semialgebraic games	
25.		TBA / Review / Additional topics	
		Project presentations	

References

[BPT12] G. Blekherman, P. A. Parrilo, and R. Thomas, editors. *Semidefinite optimization and convex algebraic geometry*, volume 13 of *MOS-SIAM Series on Optimization*. SIAM, 2012.

[BTN01] A. Ben-Tal and A. Nemirovski. *Lectures on modern convex optimization*. MPS/SIAM Series on Optimization. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2001.

[BV04] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.

[CLO97] D. A. Cox, J. B. Little, and D. O'Shea. *Ideals, varieties, and algorithms: an introduction to computational algebraic geometry and commutative algebra*. Springer, 1997.

[de 02] E. de Klerk. *Aspects of Semidefinite Programming: Interior Point Algorithms and Selected Applications*, volume 65 of *Applied Optimization*. Kluwer Academic Publishers, 2002.

[Mis93] B. Mishra. *Algorithmic Algebra*. Springer-Verlag, 1993.

[Stu02] B. Sturmfels. *Solving Systems of Polynomial Equations*. AMS, Providence, R.I., 2002.

[WSV00] H. Wolkowicz, R. Saigal, and L. Vandenberghe, editors. *Handbook of Semidefinite Programming*. Kluwer, 2000.

[Yap00] C. K. Yap. *Fundamental problems of algorithmic algebra*. Oxford University Press, New York, 2000.